

Edexcel International AS/A Level

Maths

Getting Ready to Teach

Event Code:

First teaching in 2018, first assessment 2019



Your Online Environment

- . Technical Difficulties & Support
- . Recording
- . Communication in an online environment
- . Asking Questions
- . Using Polls
- . Downloading Documents

Session Agenda

08:00 Introduction and overview

08:10 P1

08:25 P2

08:50 Activity and short break

08:55 P3

09:30 P4

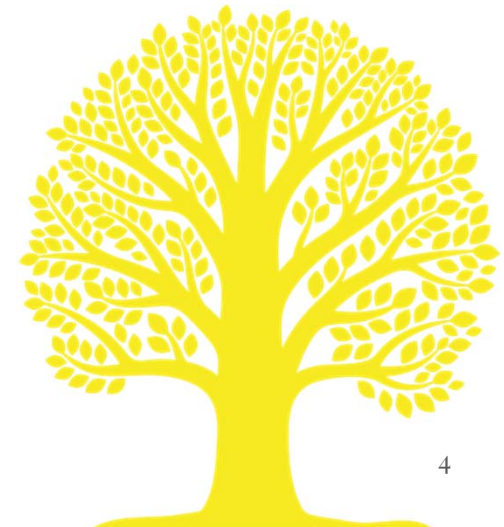
09:50 D1

09:55 Issues arising



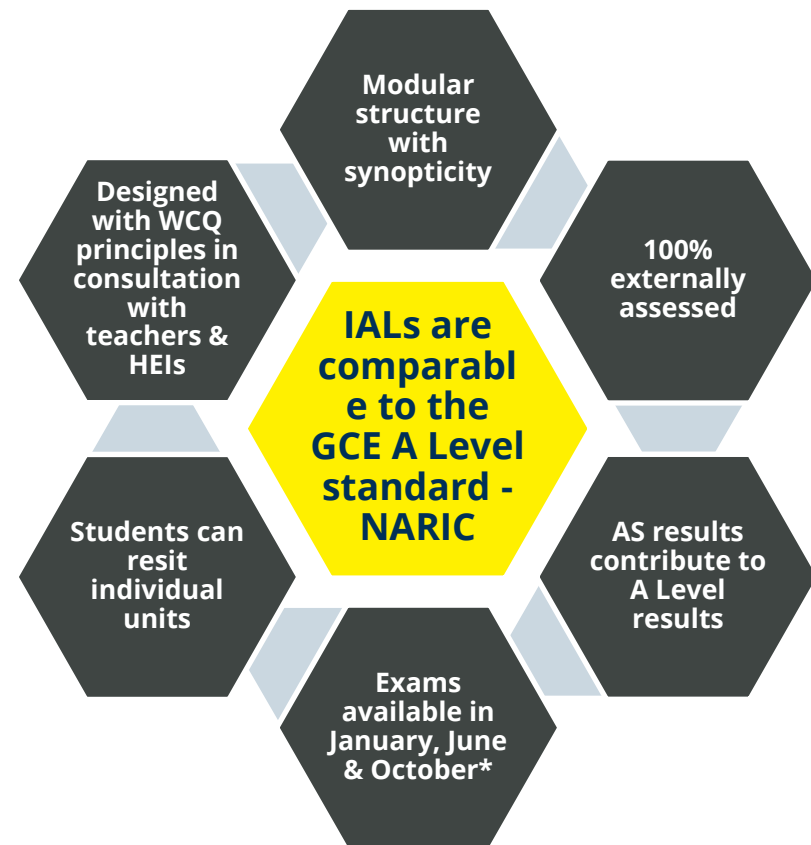
Aims and objectives

- Get an overview of the main changes in the new specification
- Explore possible teaching and learning strategies that may be employed for the new specification
- Look at Sample Assessments and Mark Schemes
- Look at planning and organisation for the new specification
- Explore the support and resources available from Pearson to guide you through teaching the new specification .



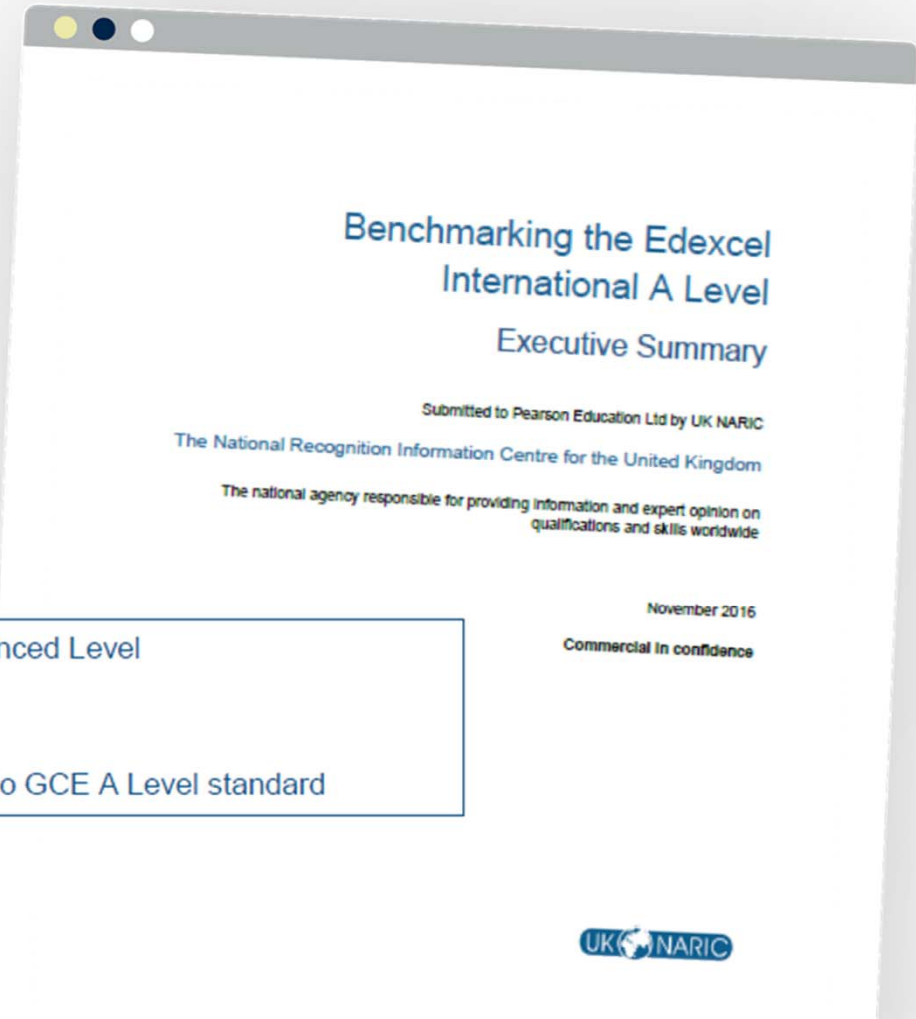
IAL Features

- International A Levels and AS Levels are created for International Students
- Globally recognised.



Updated NARIC report for Edexcel IAL

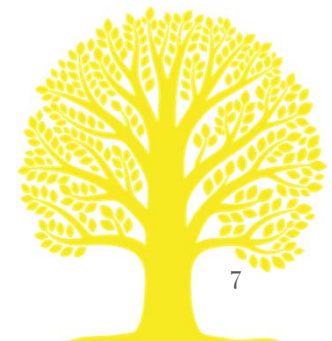
The executive summary confirms that Edexcel IALs are considered comparable to the GCE A Level standard following reforms to the UK regulated qualifications.



Qualification:	Edexcel International Advanced Level
Awarding Institution:	Pearson Education Ltd
Comparability:	Is considered comparable to GCE A Level standard

IAS & IAL subjects

Biology	Chemistry	Physics	Mathematics	Further Mathematics
Pure Mathematics	Information Technology	Business	Economics	Accounting
English Language	English Literature	History	Geography	Psychology
Arabic	French	German	Greek	Spanish
		Law (IAL only)		



World-class qualifications

All Edexcel qualifications are developed to meet Pearson's **World Class Qualification design principles**

Endorsement of educational **thought-leaders and assessment experts** from across the globe



Developed using an understanding and benchmarking of **all educational systems**

Qualifications that support young people to **develop the capabilities** they need to **progress** and prosper in their lives

IAL 2018 Mathematics

Mathematics | Further Mathematics | Pure Mathematics

**Reviewed and
updated in light
of GCE A level
changes**

**Pure Mathematics
content in 4 units**

**5 optional routes
to achieve
qualification**

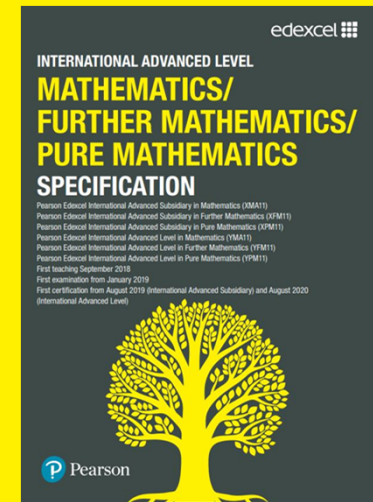
**14 equally
weighted units**

**Transferable Skills
embedded**

**Fully modular
Examinations
three times a year
AS contributes to
A level**

**Dedicated
textbooks
available and
more currently in
production**

**[TeachingMaths
@pearson.com](mailto:TeachingMaths@pearson.com)**



Polls to get to know the delegates.



Structure of the qualification

- IAL AS Mathematics
 - Compulsory units
 - P1 (WMA11/01) P2 (WMA12/01)
 - (each 90 minutes) (each 75 marks)
 - Plus **one** of the following:
 - D1 (WDM11/01), M1 (WME01/01), S1(WST01/01)
 - (each 90 minutes) (each 75 marks)
- Requires knowledge of P1, P2 and 2D vectors



Structure of the qualification

- **IAL A2 Mathematics**
 - Compulsory modules
 - P1 (WMA11/01) P2 (WMA12/01)
 - P3 (WMA13/01) P4 (WMA14/01)
 - (each 90 minutes) (each 75 marks)
 - Plus **one** of the following combinations:
 - D1 + M1 or D1 + S1 or M1 + S1
 - or M1 + M2 or S1 + S2
 - (each 90 minutes) (each 75 marks)



Structure of the qualification

- The course of study can be taught and assessed as:
 - Distinct units with assessments taken at appropriate stages
 - Or**
 - A linear course assessed in its entirety at the end.

Assessments will be generally available in Jan, June and Oct

First assessments opportunities:

P1	Jan 2019
P2, M1, S1, D1, FP1	June 2019
P3	Jan 2020
P4, M2, S2 FP2, FP3, M3, S3	June 2020



Structure of the qualification

- All units allow the use of a suitable calculator.

Calculators must not:

- be designed or adapted to offer any of these facilities:
 - language translators
 - symbolic algebraic manipulation
 - symbolic differentiation or integration
 - communication with other machines or the internet
- be borrowed from another candidate during an examination for any reason*
- have retrievable information stored in them. This includes:
 - databanks
 - dictionaries
 - mathematical formulae
 - text.

However, an invigilator
can give a candidate a
calculator



Structure of the qualification

- Why has the course changed?

The **content** has changed **slightly** in response to new thinking in the UK, especially in terms of:

Realising the importance of proof in mathematics

The assessment design has changed in **response to our centres**

Centres can decide whether to treat the course as linear or modular.
There are now 3 opportunities in the year to take most units.



Structure of the qualification

- Proof lies at the very heart of 'pure' mathematics. Students should be taught to appreciate what a proof means in mathematics.
- The 2018 course now allows **all** students to experience the power of deductive proof.
- P2 – Section 1 - AS
- P4 – Section 1 - A

So a notion of proof is
now required at AS



Structure of the qualification

The amount of change in content of the new, 2018, course is **relatively modest**.

There are NO changes in the content of M1, S1, M2, S2, M3, S3, FP1, FP2 and FP3.

There are a **small number** of changes in the content of D1.

We shall look
at this briefly
at the end



Problems and examples in this presentation

Edexcel exam questions undergo a rigorous process before any student sees the examination paper.

In several slides in this presentation the language and style are not fully that of the exams – indeed there are some problems that would not do at all as exam questions but do have a use as a teaching application.

The questions themselves are indicative also of the range that students should see in class. They are not intended in any way as a ‘pointer’ to examination questions.

The Edexcel team will be producing material which teachers will be able to use to support their teaching – especially of the new topics.



Activities in this presentation

- There are several activities in this presentation.
- Some are as material for delegates to engage with some mathematics, which may be unfamiliar.
- In all of the activities delegates are encouraged to consider (with colleagues) issues such as:
 - Alternative methods of solution
 - Teaching implications
 - Demand
- How activities/tasks/questions could be adapted.

The activities have been adapted from a face to face presentation – which is a full day session, so they are NOT intended to be completed in these two hours. But we think they may be useful for delegates to look at after this online session.



P1



Introduction to the Assessment P1

Content

Algebra and functions
Coordinate geometry in the (x, y) plane
Trigonometry
Differentiation
Integration

Assessment Objectives / Skills Tested

AO1 recall, select and use mathematics
AO2 construct rigorous mathematical arguments and proofs.
AO3 recall select and use standard mathematical models
AO4 comprehend mathematical contexts and arguments
AO5 use calculators and other resources

Structure of Assessment

One end of unit test
All questions compulsory
90 minutes
75 marks

P1 Content

- What's new:

1.5 Solution of quadratic equations by calculator.

Where working has to be shown, this will be signalled

1.7 Interpret linear* and quadratic inequalities graphically.

1.8 Represent linear* and quadratic inequalities graphically.

Also, clarification about:

*Linear inequalities have always been assessed in D1, but this is new to P1”

3.1 The ambiguous case of the sine rule. (Included).

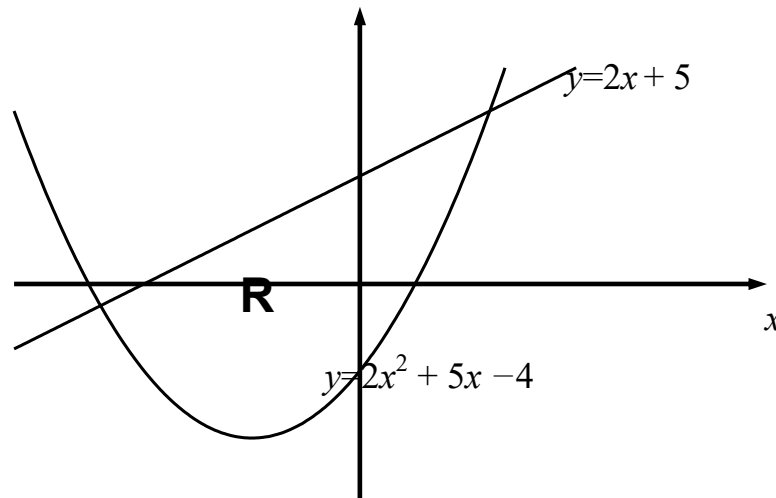
5.2 Integration of x^{-1} (specifically excluded.)



P1 Content

- What's new

1.7 Interpret linear and quadratic inequalities graphically.



R is the region bounded by the line $y = 2x + 5$ and the curve $y = 2x^2 + 5x - 4$ where the solid lines indicate that the inequalities are not strict

So $y \leq 2x + 5$ and $y \geq 2x^2 + 5x - 4$

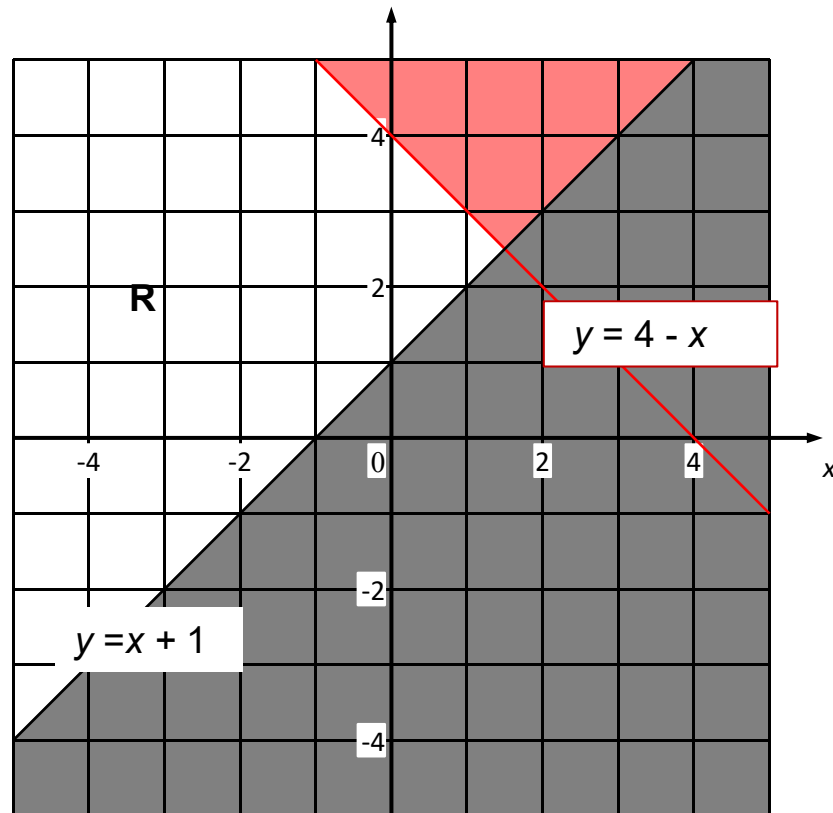
Relate this to the solution of $2x^2 + 5x - 4 \leq 2x + 5$



P1 Content

- What's new

1.8 Represent linear and quadratic inequalities graphically.



e.g. Show and label the region **R** that has all points (x, y) satisfying both the inequalities
 $y \leq 4 - x$ and
 $y \geq x + 1$

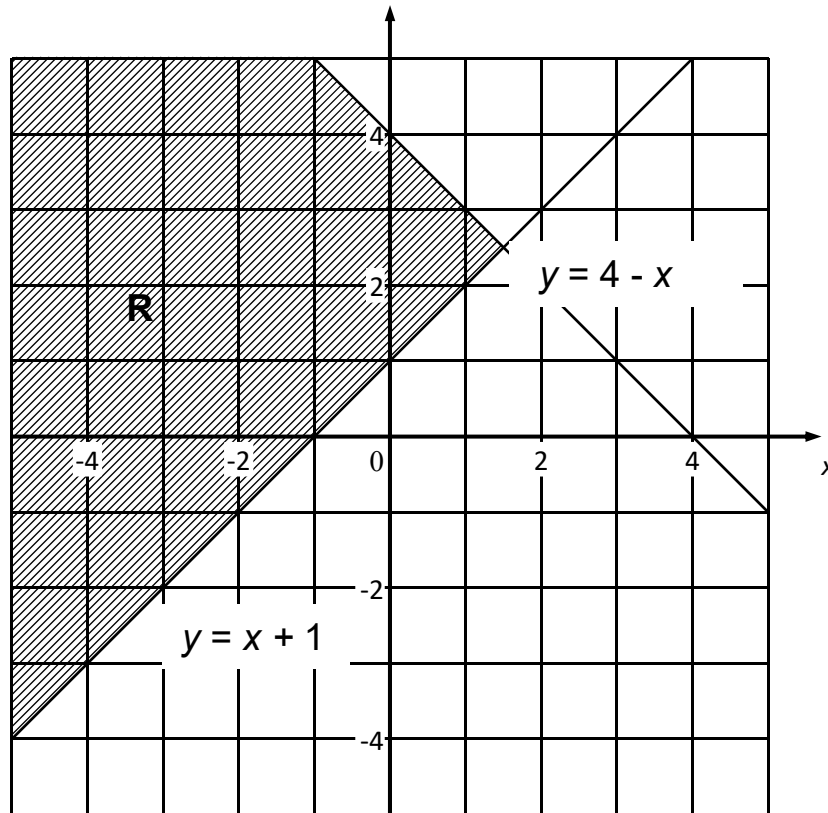
To be consistent with D1, a required region should be shown by shading out



P1 Content

- What's new

1.8 Represent linear and quadratic inequalities graphically.



However, shading in would also be acceptable.

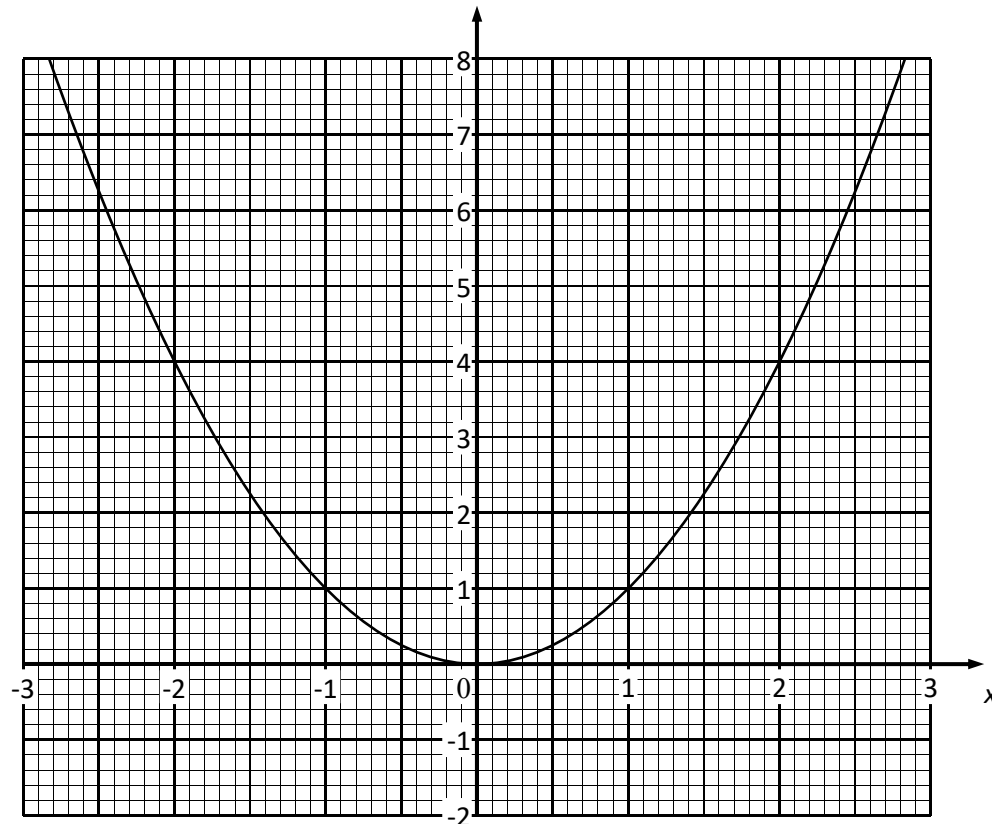
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 $y \geq x + 1$



P1 Content

- What's new?

1.8 Represent linear and quadratic inequalities graphically.

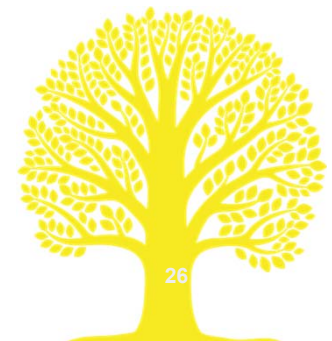


This includes inequalities with brackets and fractions. These would be reducible to linear or quadratic inequalities

An accurate graph of the curve $y = x^2$ has been drawn on the grid.

By drawing a suitable straight line on the grid solve

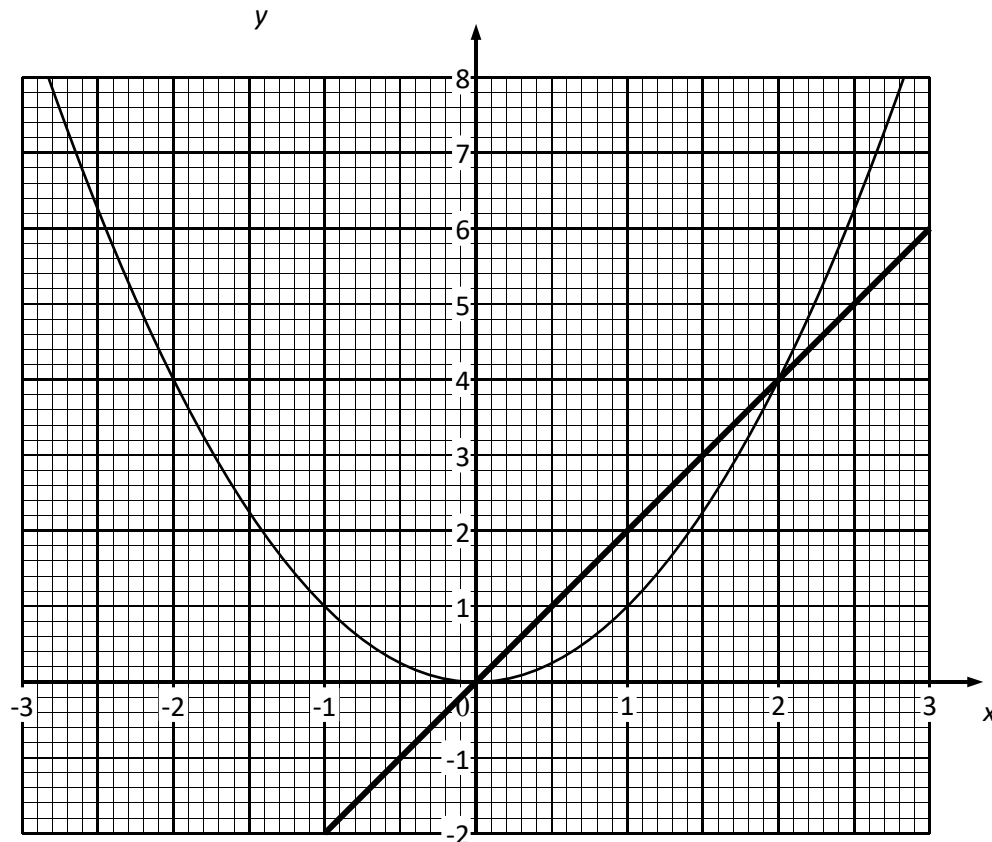
$$\frac{2}{x} < 1$$



P1 Content

- What's new?

1.8 Represent linear and quadratic inequalities graphically.



This includes inequalities with brackets and fractions. These would be reducible to linear or quadratic inequalities

By drawing a suitable straight line on the grid solve

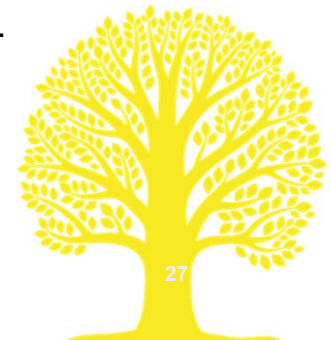
$$\frac{2}{x} < 1$$

Multiply both sides by x^2

$$2x < x^2$$

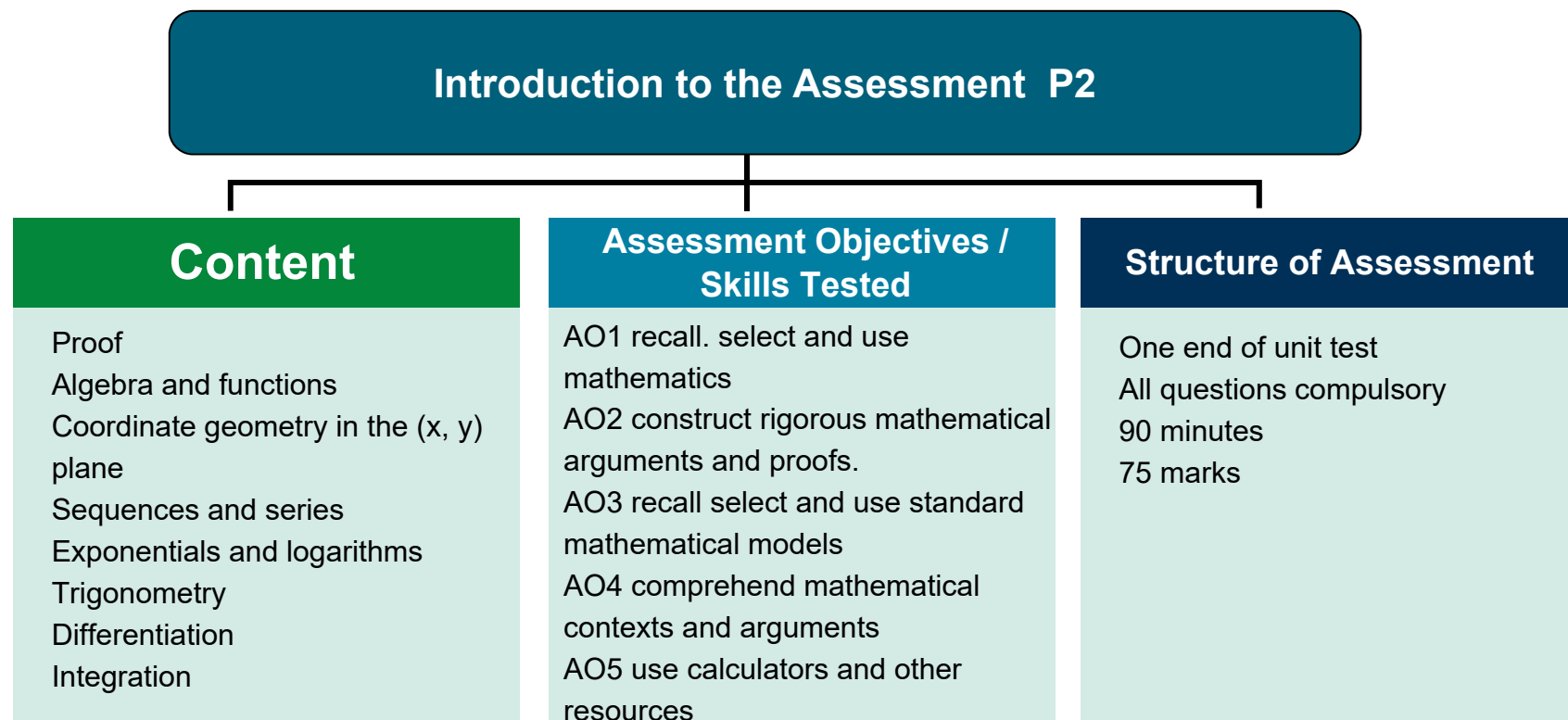
So we look at where the line is below the curve.

$$x < 0 \text{ or } x > 2$$



P2





Knowledge of the contents of P1 is required and may be tested

P2 Content

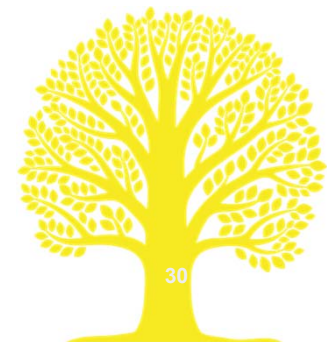
- What's new

2.1 Simple algebraic division, use of Factor Theorem and Remainder Theorem.

Now extended to divisors of the form $(ax + b)$ and $(ax - b)$

4.3 Increasing sequences, decreasing sequences and periodic sequences

4.4 Use of logs to find the value of n given the sum of a geometric series (**now explicit**)



P2 Content

- **What's new in P2**

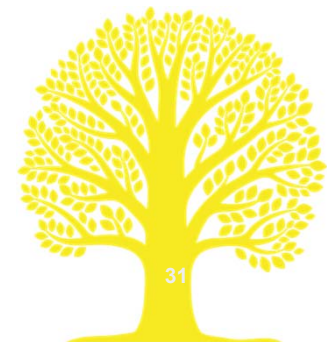
1.1 Proof; understand and use the structure of mathematical proof proceeding from given assumptions through a series of logical steps to a conclusion.

1.2 Proof by exhaustion

1.3 Disproof by counter example.

8.2 Find, using integration, the area between two curves.

8.3 Use of increasing number of trapezia to improve accuracy and an estimate of the error may be required.



P2 Content

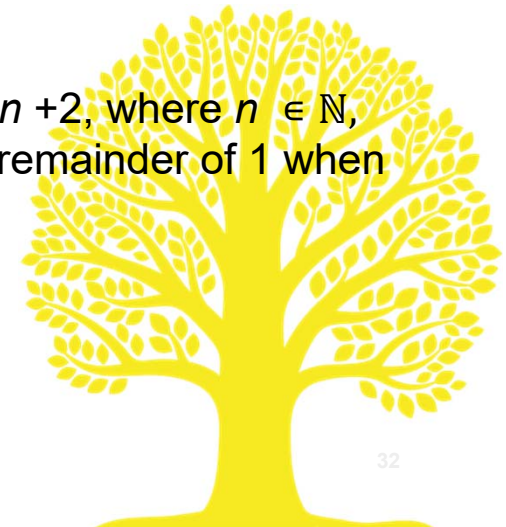
- Exploring the proof section
1.1 Proof; understand and use the structure of mathematical proof proceeding from given assumptions through a series of logical steps to a conclusion.

Both the proof of the sum of an arithmetic series and the sum of a geometric series have always been explicitly mentioned in the C12specification.....

Proof of the Factor Theorem and the Remainder Theorem are not – so should the proofs be taught?

Proofs that build on work done for IGCSE, for example

Given that any natural number can be written as $3n$ or $3n + 1$ or $3n + 2$, where $n \in \mathbb{N}$, show that any square number is either a multiple of 3 or leaves a remainder of 1 when divided by 3



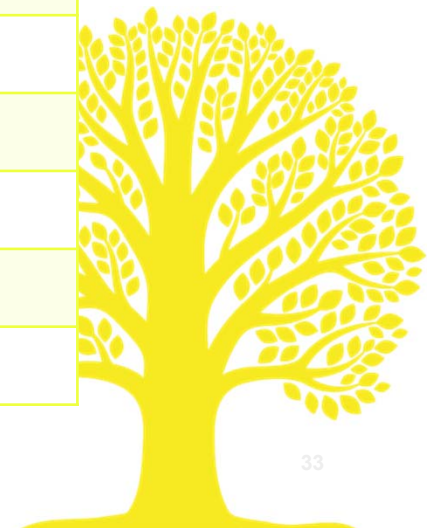
P2 Content

- Exploring the proof section
1.2 Proof by exhaustion.

Usually, but not always deals with trying all the options explicitly.

Given that x, y are odd positive integers less than 7, show their sum is always even

x	y	$x + y$	Even?
1	1	2	✓
1	3	4	✓
1	5	6	✓
3	3	6	✓
3	5	8	✓
5	5	10	✓



P2 Content

- Exploring the proof section
1.2 Proof by exhaustion.

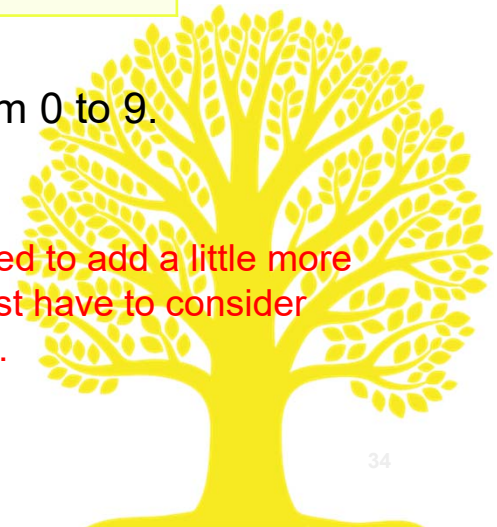
Here is a more sophisticated example.

Show that no square number has a units digit of 3

n	0	1	2	3	4	5	6	7	8	9
n^2	0	1	4	9	(1)6	(2)5	(3)6	(4)9	(6)4	(8)1

The table shows the units digit of the squares of the numbers from 0 to 9.
We can use this to prove the statement.

We would need to add a little more
on why we just have to consider
the units digit.



P2 Content

- Exploring the proof section
1.2 Proof by exhaustion.

From the specimen paper .

Prove, by exhaustion that $n^2 + 2$ is not divisible by 4 ($n \in \mathbb{N}$)

One possible approach is as follows.

If n is odd then n^2 is odd and so $n^2 + 2$ is odd so cannot be divisible by 4

If n is even then n^2 is a multiple of 4 so $n^2 + 2$ leaves a remainder of 2 when divided by 4

Study the specimen paper solution
to see how the marks are awarded



P2 Content

- Exploring the proof section (just for interest)
1.2 Disproof by counter example..

This involves disproving a 'theorem' by providing at least one case where the theorem is shown to be untrue.

Consider $P(n) = n^2 + n + 11$

n	0	1	2	3	4	5	6	7	8
P(n)	11	13	17	23	31	41	53	67	83

All the values of $P(n)$ in the table are prime. So we make the following claim:

$P(n)$ is a prime number for all $n \in \mathbb{N}$

Give a value of n for which $P(n)$ is not prime, thus refuting the claim.



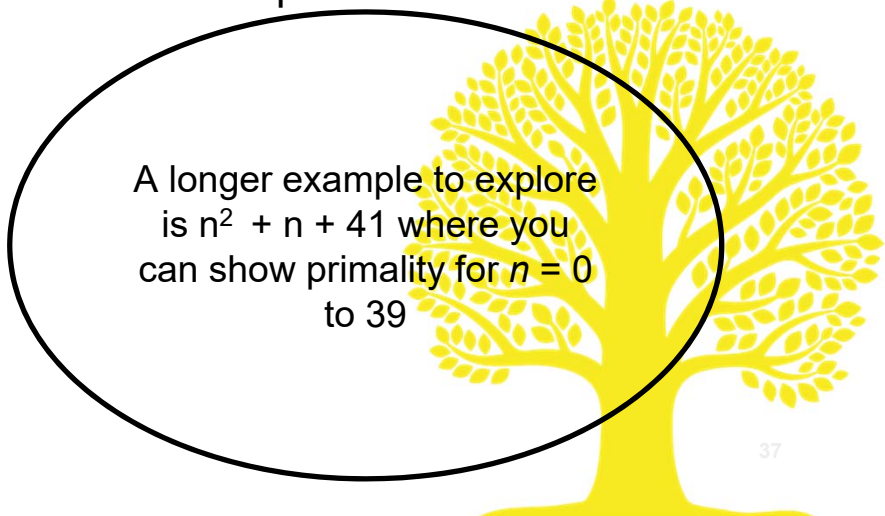
P2 Content

- Exploring the proof section (just for interest)
1.2 Disproof by counter example.

To set this as a class example we could start with agreeing $P(0)$ to $P(8)$ are prime, then

(a) Factorise $n^2 + n$

(b) Hence, find a value of n for which $n^2 + n + 11$ is NOT a prime number



A longer example to explore
is $n^2 + n + 41$ where you
can show primality for $n = 0$
to 39

P2 Content

- Activity 1

Activity 1 is a sheet of questions related to proofs .

Think about questions 3 and 11

3. Factorise $x^2 - y^2$

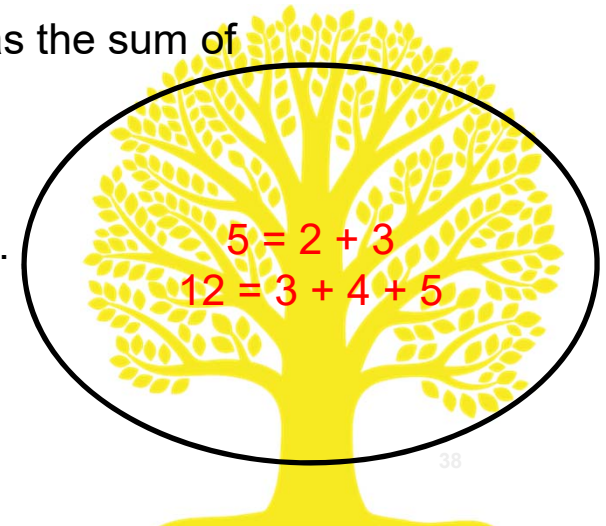
Show that any odd number can be written the difference of the squares of consecutive numbers.

e.g. $7 = 4^2 - 3^2$, $55 = 28^2 - 27^2$

11 I claim that any natural number greater than 1 can be written as the sum of consecutive natural numbers.

Give a counterexample to show my claim is wrong.

Suggest a different claim based on (at least 2!) counterexamples.



P2 Content

- Activity 1

3. Factorise $x^2 - y^2$

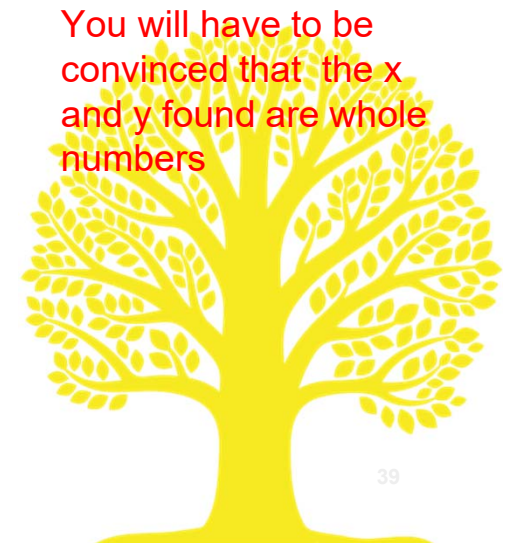
Show that any odd number can be written the difference of the squares of consecutive numbers.

e.g. $7 = 4^2 - 3^2$, $55 = 28^2 - 27^2$

$x^2 - y^2 = 1 \times N$ so let $x - y = 1$ and $x + y = N$ and solve for x and y in terms of N

11 2, 4 and 8 can be quickly found to be counter examples.

You will have to be
convinced that the x
and y found are whole
numbers



P2 Content

2.1 Simple algebraic division, use of Factor Theorem and Remainder Theorem.

Now extended to divisors of the form $(ax + b)$ and $(ax - b)$

(a) Use the factor theorem to show that $(3x - 4)$ is a factor of

$$P(x) = 3x^3 - 10x^2 - x + 12$$

Hence or otherwise ,

(b) Factorise $= 3x^3 - 10x^2 - x + 12$

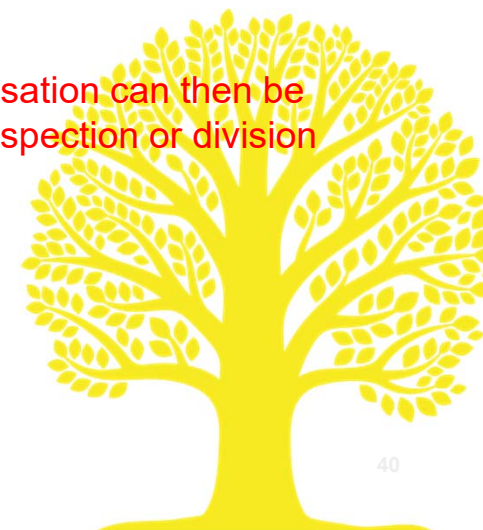
$$P\left(\frac{4}{3}\right) = \frac{64 - 160 - 12}{9} + 12 = 0$$

Therefore, $(3x - 4)$ is a factor

$$P(x) = (3x - 4)(x^2 \dots - 3) = (3x - 4)(x^2 - 2x - 3)$$

$$P(x) = (3x - 4)(x - 3)(x + 1)$$

Full factorisation can then be done by inspection or division



P2 Content

4.3 Increasing sequences, decreasing sequences and periodic sequences

A sequence u_n is increasing if $u_{n+1} > u_n$ for all n

So 1, 3, 5,

A sequence u_n is decreasing if $u_{n+1} < u_n$ for all n

So 100, 50, 25,

A sequence u_n is periodic if $u_{n+k} = u_n$ for all n

k will be the period if it is the least number for which $u_{n+k} = u_n$



P2 Content

4.3 Increasing sequences, decreasing sequences and periodic sequences

A sequence u_n is periodic (of period k) if $u_{n+k} = u_n$ for all n

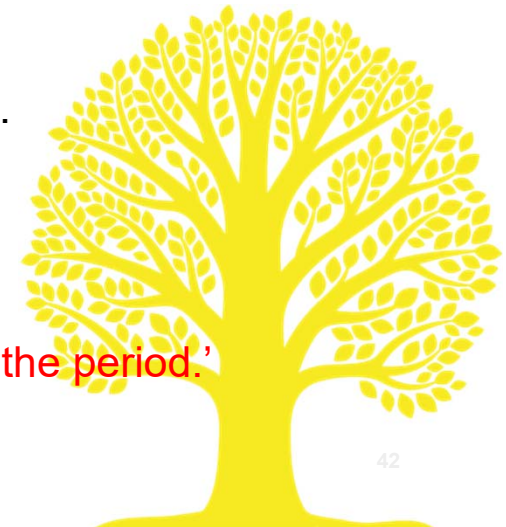
The sequence $u_{n+1} = \frac{1}{u_n}$ with $u_1 = u \ (\neq 0)$ has $u, 1/u, u, \dots$ is periodic ($k = 2$)

The sequence $u_{n+1} = 7 - u_n$ is periodic ($k = 2$) when $u_1 = u$

The sequence $u_n = \sin \frac{2n\pi}{3}$ is periodic ($k = 3$), $n = 1, 2, 3, \dots$

You might like to think about this question later

'Show that the sequence $u_{n+1} = a - u_n$ is periodic and state the period.'



P2 Content

4.4 Use of logs to find the value of n given the sum of a geometric series (now explicit)

This is not so much new as an important use of logs that has been a weakness – see previous examiner reports and Edexcel feedbacks.

The first term of a geometric series is 6 and the common ratio is 0.92.

June 2016 C12 Q9

The sum to n terms of this series is greater than 72.

(b) Calculate the smallest possible value of n . **(4)**

$$6 \frac{(1 - 0.92^n)}{1 - 0.92} > 72 \Rightarrow 0.92^n < 0.04$$

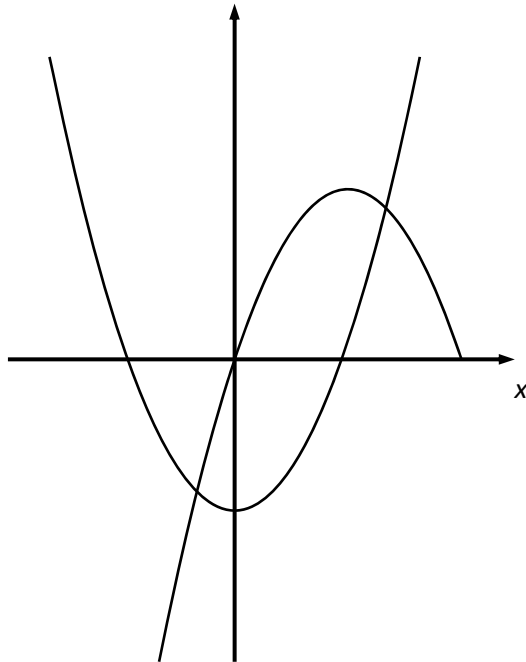
$$n \log 0.92 < \log 0.04$$

$$n > \frac{\log 0.04}{\log 0.92} = 39$$



P2 Content

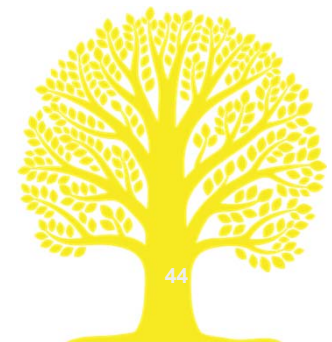
8.2 Find, using integration, the area between two curves



The sketch shows parts of the graphs of

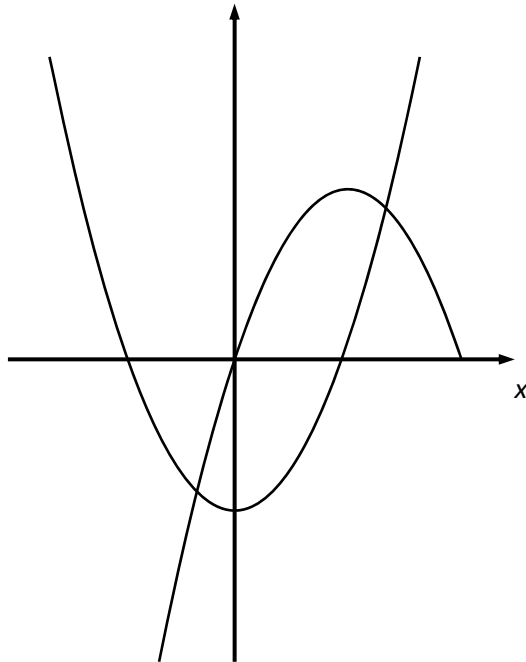
$$y = 3x - x^2 \text{ and}$$
$$y = x^2 - 2$$

Find, using calculus, the area of the finite region bounded by the two curves.



P2 Content

8.2 Find, using integration, the area between two curves

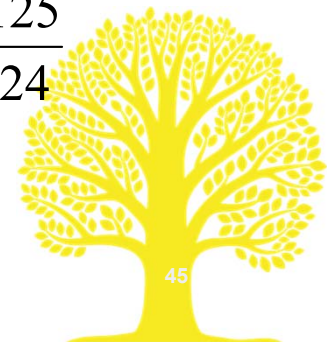


How could we structure this question to make it (i) more accessible
(ii) less accessible?

The curves intersect at the solutions
 $3x - x^2 = x^2 - 2 = y$

$$x = 2 \text{ or } x = -\frac{1}{2}$$

$$\begin{aligned} & \int_{-\frac{1}{2}}^2 (3x - x^2 - [x^2 - 2]) dx \\ &= \int_{-\frac{1}{2}}^2 (2 + 3x - 2x^2) dx \\ &= \left[2x + \frac{3}{2}x^2 - \frac{2}{3}x^3 \right]_{-\frac{1}{2}}^2 = \frac{125}{24} \end{aligned}$$



P2 Content

8.2 Find, using integration, the area between two curves

In the Specimen paper, the task is given some structure to enable students to make progress.

Figure 2 shows a sketch of part of the curves C_1 and C_2 with equations

$$C_1: y = 10x - x^2 - 8 \quad x > 0$$

$$C_2: y = x^3 \quad x > 0$$

The curves C_1 and C_2 intersect at the points A and B .

(a) Verify that the point A has coordinates $(1, 1)$ 1 mark

(b) Use algebra to find the coordinates of the point B 6 marks

The finite region R is bounded by C_1 and C_2

(c) Use calculus to find the exact area of R 5 marks

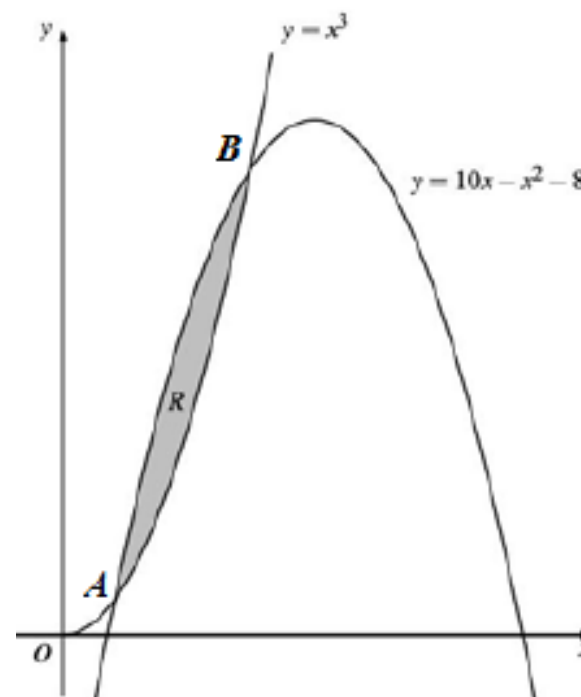
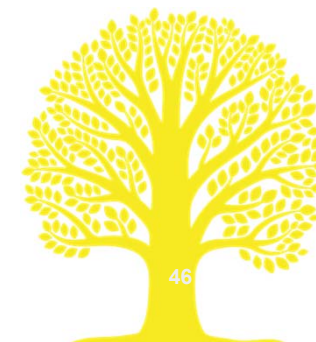


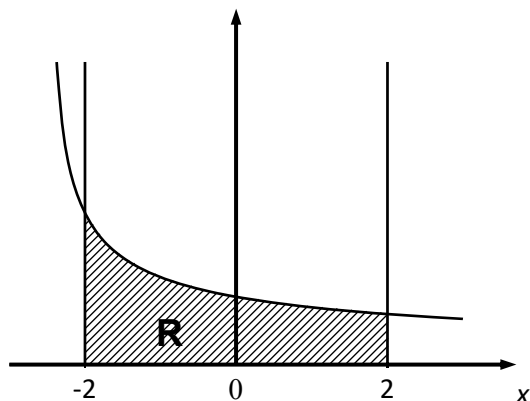
Figure 2



P2 Content

8.3 Use of increasing number of trapezia to improve accuracy and an estimate of the error may be required.

This was in C34



The figure shows a sketch of part of the curve C, with equation

$$y = \frac{1}{\sqrt{2x+5}}$$

The finite region **R** shown shaded is bounded by C, the x-axis and the lines $x = \pm 2$

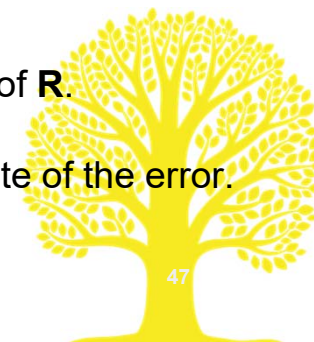
x	-2	-1	0	1	2
$y = \frac{1}{\sqrt{2x+5}}$	1		0.4472		0.3333

(a) Complete the table.

(b) Use the trapezium rule to find an estimate of the area of **R**.

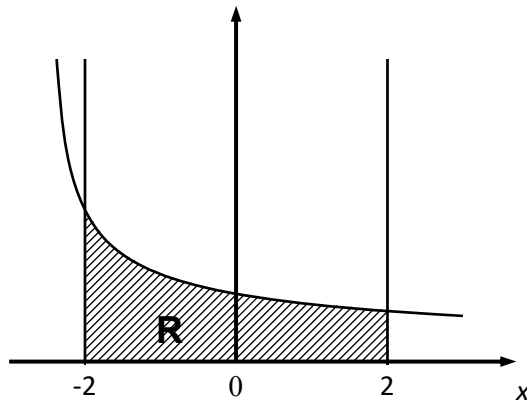
(c) Given that the exact area of **R** is 2, work out an estimate of the error.

In the exam the language is much more precise!



P2 Content

8.3 Use of increasing number of trapezia to improve accuracy and an estimate of the error may be required.



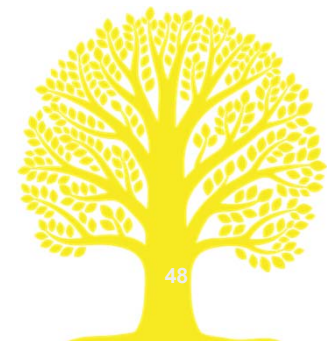
x	y	x	y
-2	1	-2	1
-1	0.57735	-1.5	0.707107
0	0.447214	-1	0.57735
1	0.377964	-0.5	0.5
2	0.333333	0	0.447214
	2.069195	0.5	0.408248
		1	0.377964
		1.5	0.353553
		2	0.333333
			2.019052

In this case doubling the number of strips reduces the error from about 3½% to about 2%

If you are faced with the question does doubling the number of strips half the error, how would you answer it?

Geometrically more strips leads to a lower error because

However, we must be careful when increasing the number of strips because.....



P2 Content

Activity 2 is a sheet of questions related to proofs .

Use the poll response to answer the questions.

As there is only a short time for this please concentrate on Q1



P3



Introduction to the Assessment P3

Content	Assessment Objectives / Skills Tested	Structure of Assessment
Algebra and functions Trigonometry Exponentials and logarithms Differentiation Integration Numerical methods	AO1 recall, select and use mathematics AO2 construct rigorous mathematical arguments and proofs. AO3 recall select and use standard mathematical models AO4 comprehend mathematical contexts and arguments AO5 use calculators and other resources	One end of unit test All questions compulsory 90 minutes 75 marks

Knowledge of the contents of P1 and P2 is required and may be tested.

The content of C12 was always required for C34, so the fact that the knowledge of P1 and P2 is needed for P3 follows the same philosophy.

P3 Content

- What's new

1.3 The use of sketch graphs as an aid to solving equations of the form $|f(x)| = g(x)$ and inequalities of the form $|f(x)| > g(x)$ where f and g are linear.

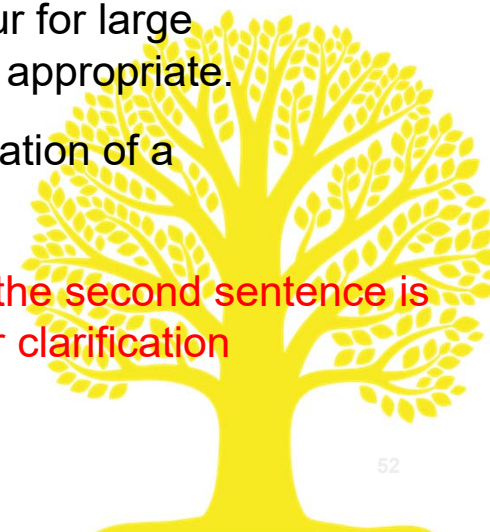
3.3 Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = ab^x$

4.1 Differentiation of e^{kx} , $\ln kx$, $\sin kx$, $\cos kx$, $\tan kx$ is an **explicit** section

4.4 Understand and use exponential growth and decay. Students should be familiar with terms such as 'initial' and be able to explore behaviour for large values of t or to consider whether the range of values predicted is appropriate.

4.4 Understand and use exponential growth and decay. Consideration of a second improved model may be required.

For 4.4 the second sentence is there for clarification



P3 Content

- What's new

5.1 Integration of e^{kx} , $\sin kx$, $\cos kx$ and $1/x^n$ **explicitly** given

5.2 Integration by recognition of known derivatives

$$\left| \int f'(x)[f(x)]^n dx = \frac{[f(x)]^{n+1}}{n+1} + c \right.$$



P3 Content

- What's new
1.3 The modulus function. The use of sketch graphs as an aid to solving equations of the form $|f(x)| = g(x)$ and inequalities of the form $|f(x)| > g(x)$ where f and g are linear.

(a) Sketch the graphs of $y = |2x - 3|$ and $y = x + 1$ on the same axes.

(b) Use your sketch to solve $|2x - 3| < x + 1$

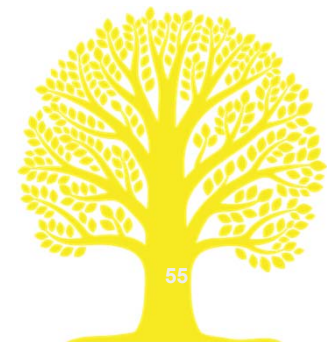
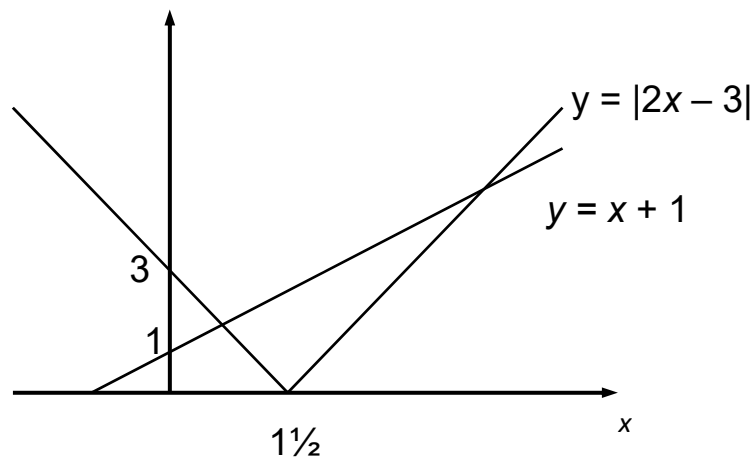
(c) For the equation $|2x - 3| = kx + 1 \quad k \in \mathbb{R}$

what is the relationship between the number of real solutions for x and the value of k ?



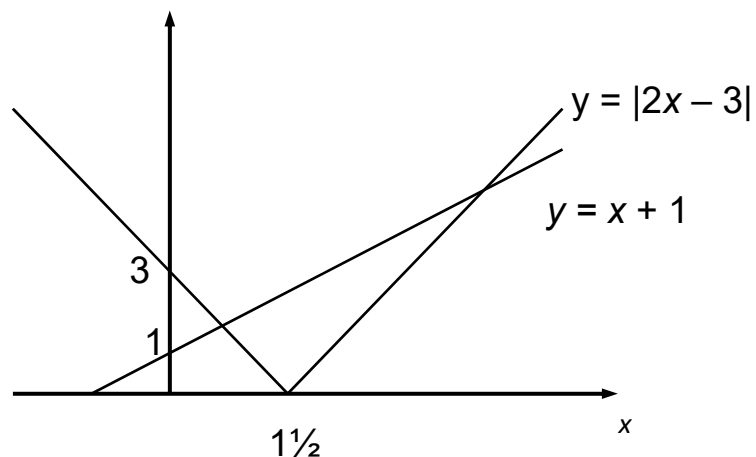
P3 Content

- What's new
 - (a) Sketch the graphs of $y = |2x - 3|$ and $y = x + 1$ on the same axes.
 - (b) Use your sketch as an aid to solve $|2x - 3| < x + 1$



P3 Content

- What's new



Possible approaches for (b)?

Mine is for $x > 1.5$ we consider

$$x + 1 = 2x - 3 \Rightarrow x = 4 \quad \checkmark$$

For $x < 1.5$ we consider

$$x + 1 = -(2x - 3) \Rightarrow x = \frac{2}{3} \quad \checkmark$$

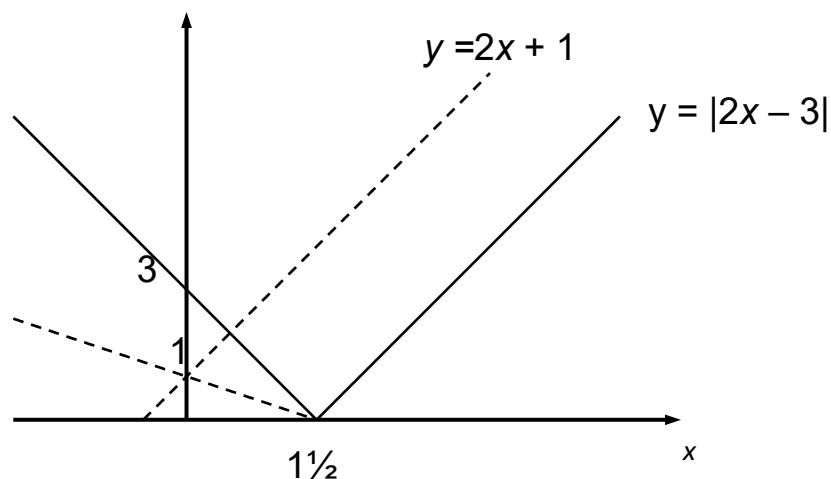
$$\text{So } \frac{2}{3} < x < 4$$



P3 Content

- What's new

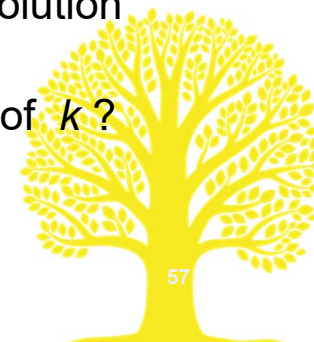
- (c) For the equation $|2x - 3| = kx + 1$ $k \in \mathbb{R}$
what is the relationship between the number of real solutions and the value of k ?



I've marked some relevant straight lines which have a bearing on the question.

So for $k > 2$ there is 1 real solution

What about the other values of k ?



P3 Content

- What's new

3.3 Use logarithmic graphs to estimate parameters in relationships of the form

$$y = ax^n \quad \text{and} \quad y = ab^x$$

$$y = ax^n$$

$$\log y = \log a + n \log x$$

$$Y = \log a + nX$$

Gives a straight line graph in the (X, Y) plane with gradient n and intercept on the y-axis of $\log a$

My scientist colleagues seem to prefer common to natural logs

What are the teaching issues, if any, with using common instead of natural logarithms?

$$y = ab^x$$

$$\log y = \log a + x \log b$$

$$Y = \log a + x \log b$$

Gives a straight line graph in the (x, Y) plane with gradient $\log b$ and intercept on the Y-axis of $\log a$



P3 Content

Power laws are common in nature

- What's new

3.3 Use logarithmic graphs to estimate parameters in relationships of the form
 $y = ax^n$

A company make models in different sizes of an iconic building.

The table gives information about the costs and heights of the models.

Height(h cm)	10	15	20	30	50
Cost (\$ y)	3	8	20	70	300

Assuming the relationship between cost (\$ y) and height (h cm) is

$$y = ah^n$$

- draw a suitable straight line graph
- Use your graph to find an estimate for n and an estimate for a .



P3 Content

- What's new

3.3 Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$

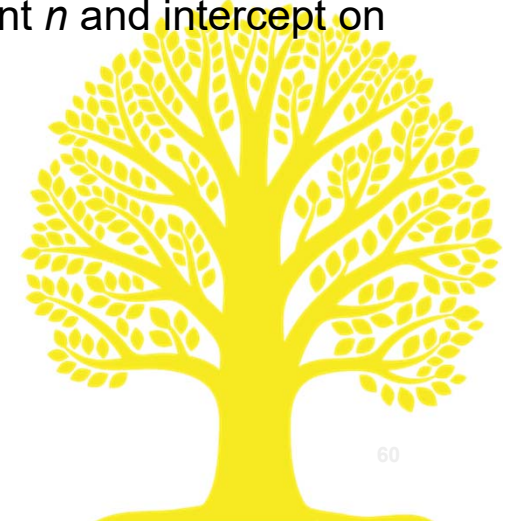
Height(h cm)	10	15	20	30	50
Cost (\$ y)	3	8	20	70	300

$$y = ah^n$$
$$\log y = \log a + n \log h$$

$$Y = \log a + nX$$

Log h	1.00	1.18	1.30	1.48	1.70
Log y	0.40	0.93	1.30	1.83	2.49

Gives a straight line graph in the (X , Y) plane with gradient n and intercept on the y -axis of $\log a$

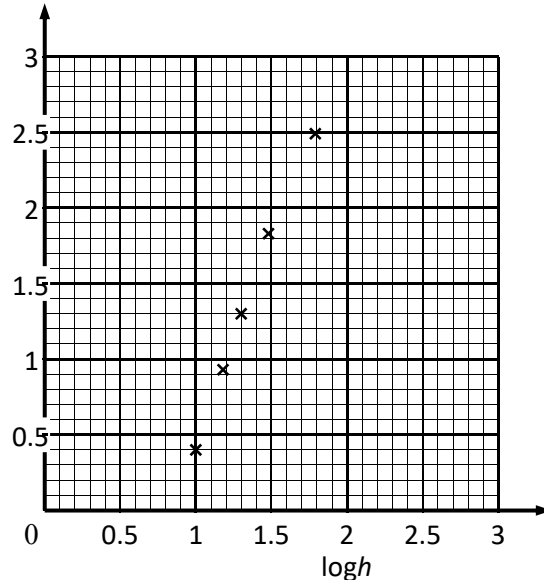


P3 Content

- What's new

3.3 Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$

Log h	1.00	1.18	1.30	1.48	1.70
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$$y = ah^n$$

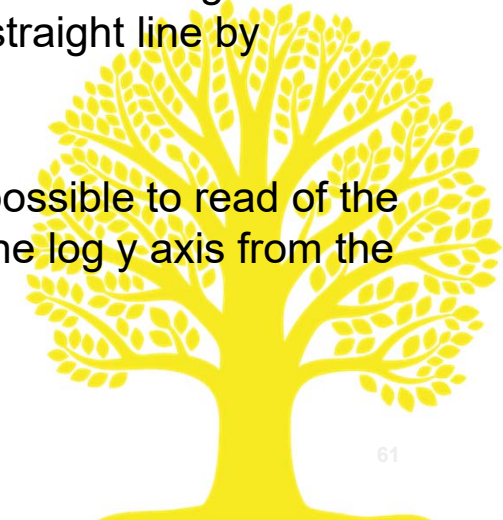
$$\log y = \log a + n \log h$$

$$Y = \log a + nX$$

Gives a straight line graph in the (X, Y) plane with gradient n and intercept on the y -axis of $\log a$

The points lie near to straight line.
Draw the best straight line by judgment

It will not be possible to read of the intercept on the $\log y$ axis from the graph.....



P3 Content

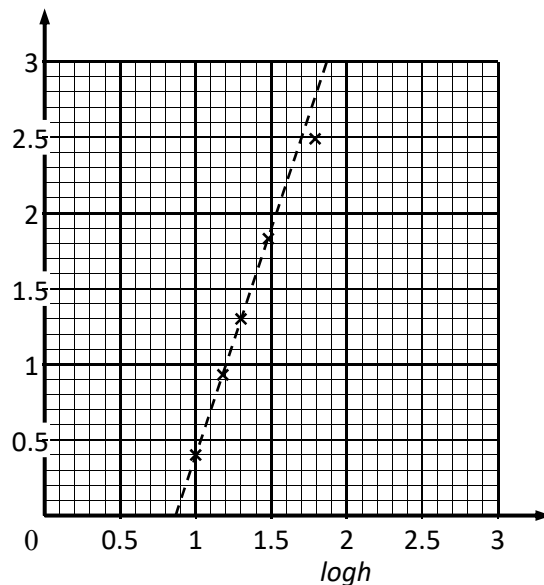
- What's new

3.3 Use logarithmic graphs to estimate parameters in relationships of the form

$$y = ax^n \quad \text{and} \quad y = ab^x$$

Log h	1.00	1.18	1.30	1.48	1.70
Log y	0.40	0.93	1.30	1.83	2.49

$$\text{Gradient} = \frac{2.5 - 0.1}{1.7 - 0.9} = 3$$



It will not be possible to read off the intercept on the log y axis from the graph

This will happen when $0 < a < 1$

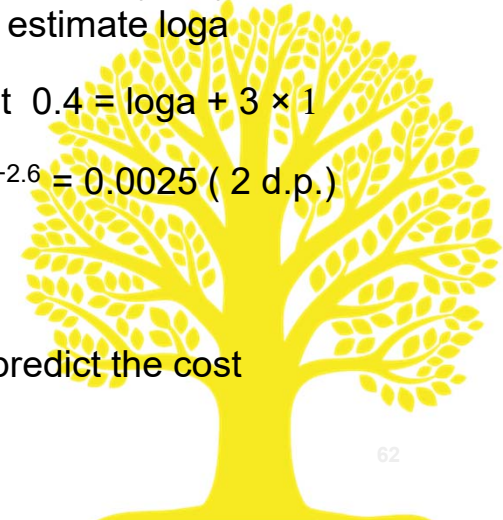
So pick a suitable value of (X, Y) on the line to calculate an estimate $\log a$

I chose $(1, 0.4)$ so that $0.4 = \log a + 3 \times 1$

$\log a = -2.6$ so $a = 10^{-2.6} = 0.0025$ (2 d.p.)

So $y = 0.0025h^3$

We then could (but shouldn't) predict the cost of a one metre tower



P3 Content

- What's new

3.3 Use logarithmic graphs to estimate parameters in relationships of the form $y = ab^x$

These are essentially exponential
increase and decrease laws

t (hours)	20	40	60	80	100	120
Mass (grams)	8.11	6.57	5.33	4.32	3.50	2.84

The table shows the mass (m grams) of radioactive molybdenum 99 in a container after t hours.

Given that the relationship is thought to be of the form $m = ab^t$ draw a suitable graph to confirm this and estimate the values of a and b

Estimate the initial mass of molybdenum 99



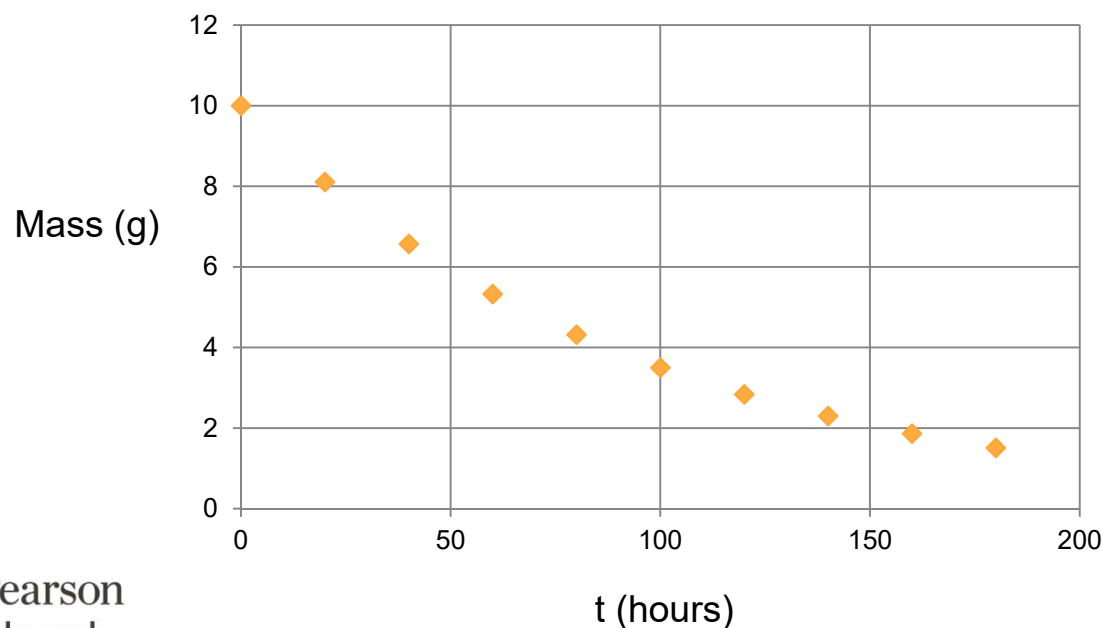
P3 Content

- What's new

3.3 Use logarithmic graphs to estimate parameters in relationships of the form $y = ab^x$

t (hours)	20	40	60	80	100	120
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The table shows the mass (m grams) of radioactive molybdenum 99 in a container after t hours.



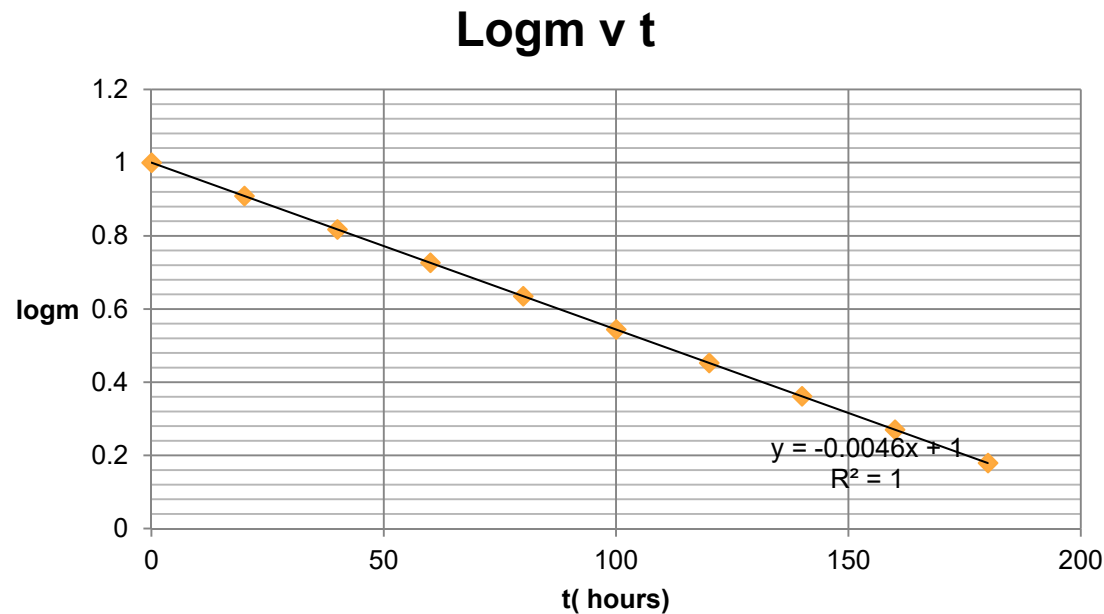
Follow a nice exponential decrease curve



P3 Content

- What's new

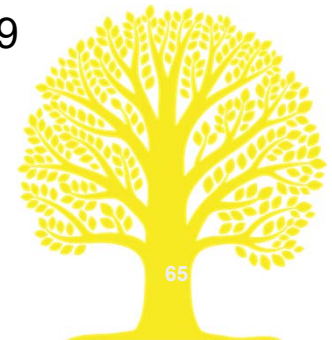
3.3 Use logarithmic graphs to estimate parameters in relationships of the form $y = ab^x$



Intercept = 1
Slope = -0.0046

$a = 10$
 $\log b = -0.0046$
 $b = 0.99$

Say 4/5 marks - 2 for the correct form to plot
- 2 for finding a and b
- 1 for accurate values of a and b



P3 Content

- What's new

4.4 Understand and use exponential growth and decay. Students should be familiar with terms such as 'initial' and be able to explore behaviour for large values of t or to consider whether the range of values predicted is appropriate.

13.



Figure 5

A colony of ants is being studied. The number of ants in the colony is modelled by the equation

$$P = 200 - \frac{160e^{0.6t}}{15 + e^{0.8t}} \quad t \in \mathbb{R}, t \geq 0$$

where P is the number of ants, measured in thousands, t years after the study started. A sketch of the graph of P against t is shown in Figure 5

Up to time T

Adapted from C34 June 2017

(a) Find the initial number of ants in the study.

(b) Show that the minimum value of P occurs at time T .

(c) What happens to the value of P as t increases beyond T ?



P3 Content

- What's new

4.4 Understand and use exponential growth and decay. Students should be familiar with terms such as 'initial' and be able to explore behaviour for large values of t or to consider whether the range of values predicted is appropriate.

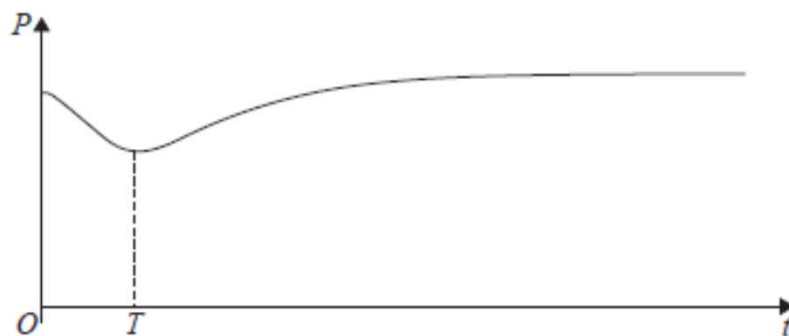


Figure 5

Taken from C34 June 2017

(a) Find the initial number of ants in the study.

$$T = 0, P = 200 - 160/(15 + 1) = 190$$

(b) Show that P has a minimum at time T .

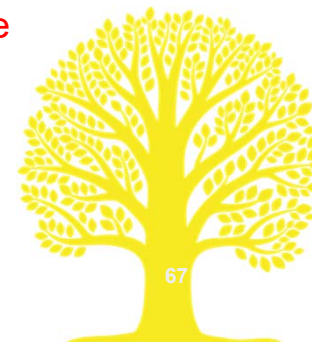
(b) Show that P has a minimum at time T .

$$\frac{dP}{dt} = -\frac{(15 + e^{0.8t}) \times 96e^{0.6t} - 160e^{0.6t} \times 0.8e^{0.8t}}{(15 + e^{0.8t})^2}$$

Taken from C34 June 2017
mark scheme

$$\text{Sets } \pm \frac{(15 + e^{0.8t}) \times 96e^{0.6t} - 160e^{0.6t} \times 0.8e^{0.8t}}{(15 + e^{0.8t})^2} = 0 \Rightarrow e^{0.8t} = 45$$

$$\Rightarrow T = \frac{\ln 45}{0.8} = 4.76$$



P3 Content

- What's new

4.4 Understand and use exponential growth and decay. Students should be familiar with terms such as 'initial' and be able to explore behaviour for large values of t or to consider whether the range of values predicted is appropriate.

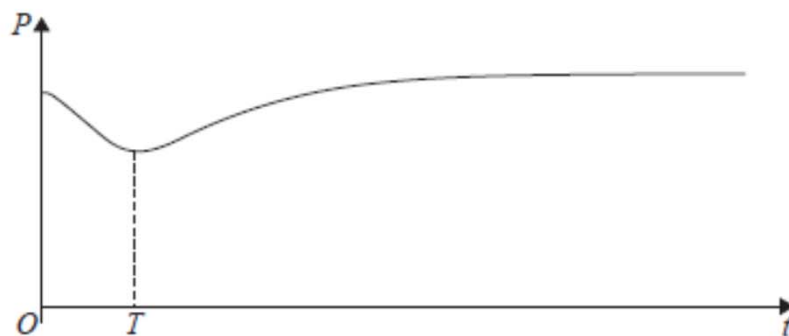


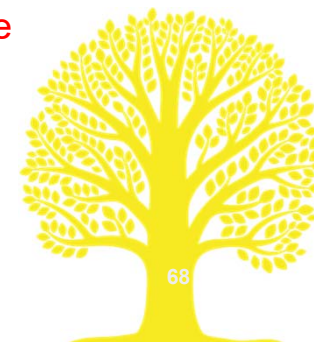
Figure 5

Taken from C34 June 2017

(c) What happens to the value of P as t increases beyond T ?

For large values of t , P behaves like $200 - 160e^{-0.2t}$ so tends towards 200 from below (as, in fact shown in the figure)

Taken from C34 June 2017
mark scheme



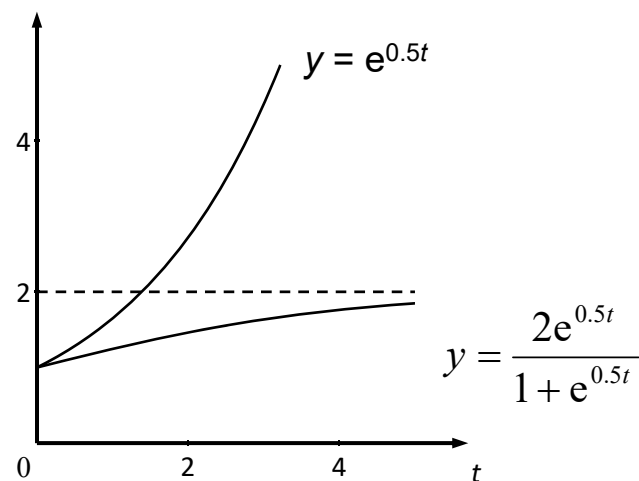
P3 Content

- What's new

4.4 Understand and use exponential growth and decay. Consideration of a second improved model may be required.

The simple model of exponential growth $N = N_0 e^{kt}$ predicts unrestricted growth as t increases.

A more sophisticated model with $k > 0$ is the 'logistic' equation
$$N = \frac{Ae^{kt}}{1 + Be^{kt}}$$



The initial value of N is $A/(1 + B)$

N tends to A/B as t gets large.



P3 Content

- What's new

5.2 Integration by recognition of known derivatives

$$\left| \begin{aligned} \int \frac{f'(x)}{f(x)} dx &= \ln f(x) + c \text{ and} \\ \int f'(x)[f(x)]^n dx &= \frac{[f(x)]^{n+1}}{n+1} + c \end{aligned} \right.$$

The first being more familiar to students than the second.

$$\int \frac{x}{x^2 + 4} dx$$

$$\int_0^{\pi/2} \frac{\sin x - \cos x}{\sin x + \cos x} dx$$



P3 Content

- What's new

5.2 Integration by recognition of known derivatives

$$\left| \begin{aligned} \int \frac{f'(x)}{f(x)} dx &= \ln f(x) + c \text{ and} \\ \int f'(x)[f(x)]^n dx &= \frac{[f(x)]^{n+1}}{n+1} + c \end{aligned} \right.$$

The first being more familiar to students than the second.

$$\int \sqrt{x+4} dx$$

$$\int x\sqrt{x^2+1} dx$$

$$\int_0^{\pi/2} \sin 2x(1+\sin^2 x)^2 dx$$



P3 Content

Activity 3

There are several questions based on P3

Please look initially at Q5 (from the specimen paper)

8. In a controlled experiment, the number of microbes, N , present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

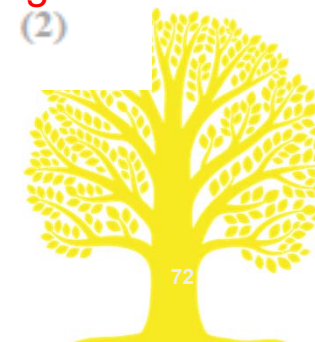
$$N = aT^b \quad \text{where } a \text{ and } b \text{ are constants}$$

- (a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b .

Scientists often like to use base 10 rather than natural logs – any thoughts?



P3 Content

Activity 3

$$N = aT^b$$

$$\log_{10} N = m \log_{10} T + c$$

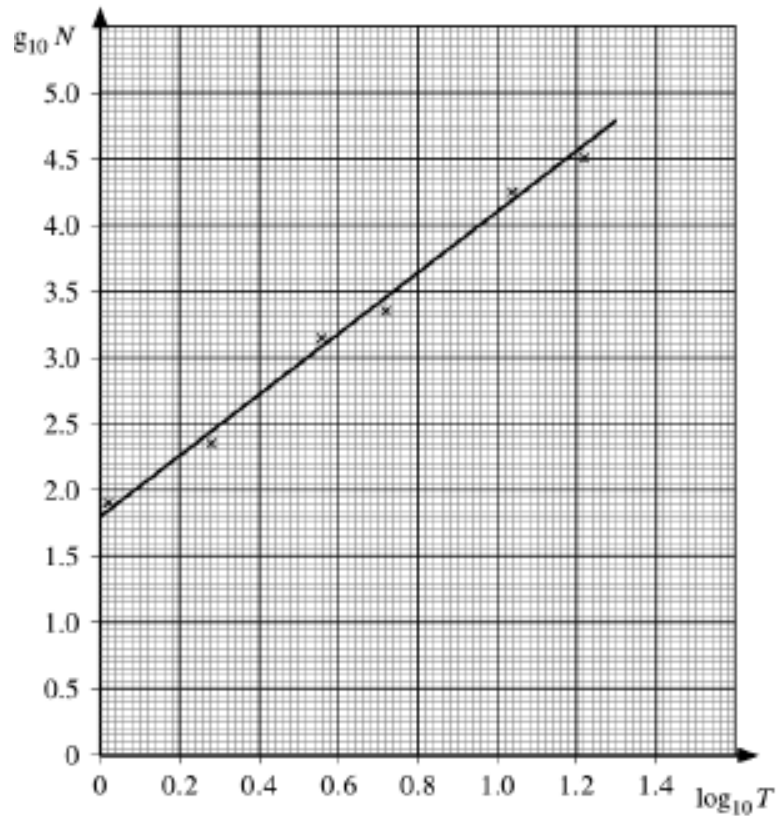


Figure 2

Use the information provided to estimate the number of microbes present 3 days after the start of the experiment

With reference to the model, interpret the value of the constant a



P3 Content

Activity 3

$$N = aT^b$$

$$\log_{10} N = m \log_{10} T + c$$

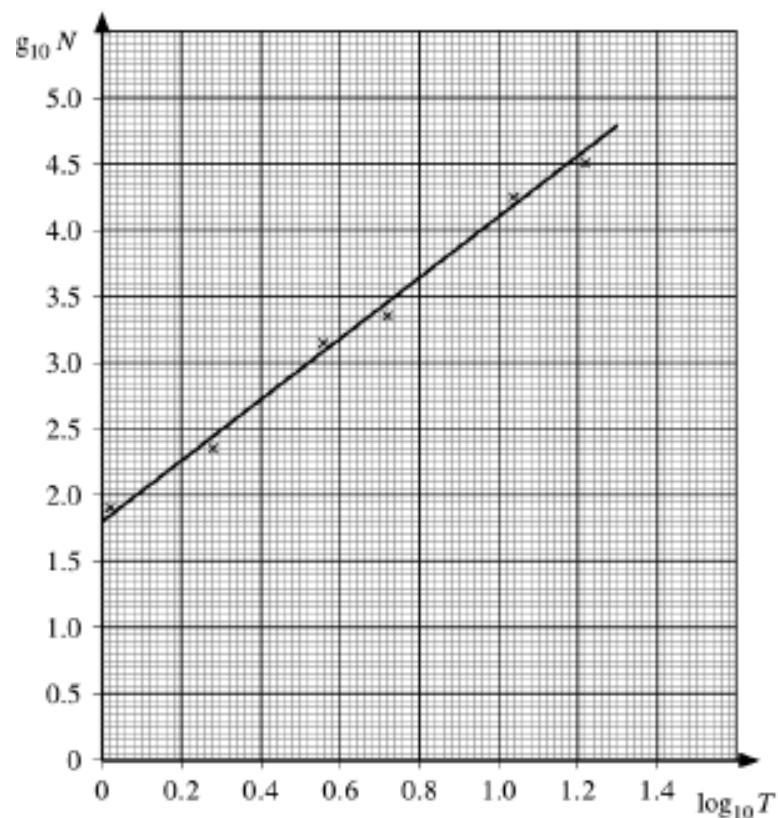


Figure 2

Use the information provided to estimate the number of microbes present 3 days after the start of the experiment

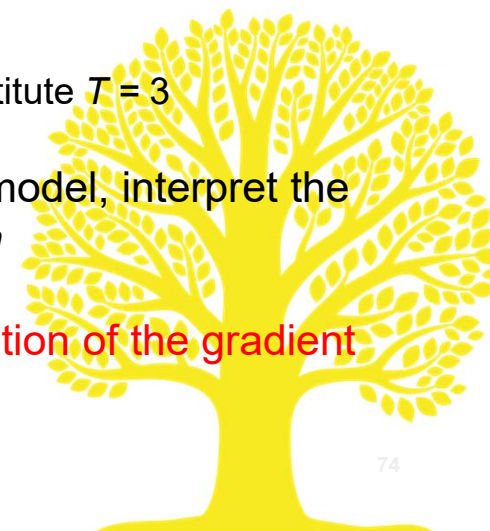
$$m \approx (4.55 - 2.25) \div (1.2 - 0.2) = 2.3$$

$$c \approx 1.85 \text{ so } a \approx 10^{1.85} \approx 71$$

$$N = 71 T^{2.3} \text{ then substitute } T = 3$$

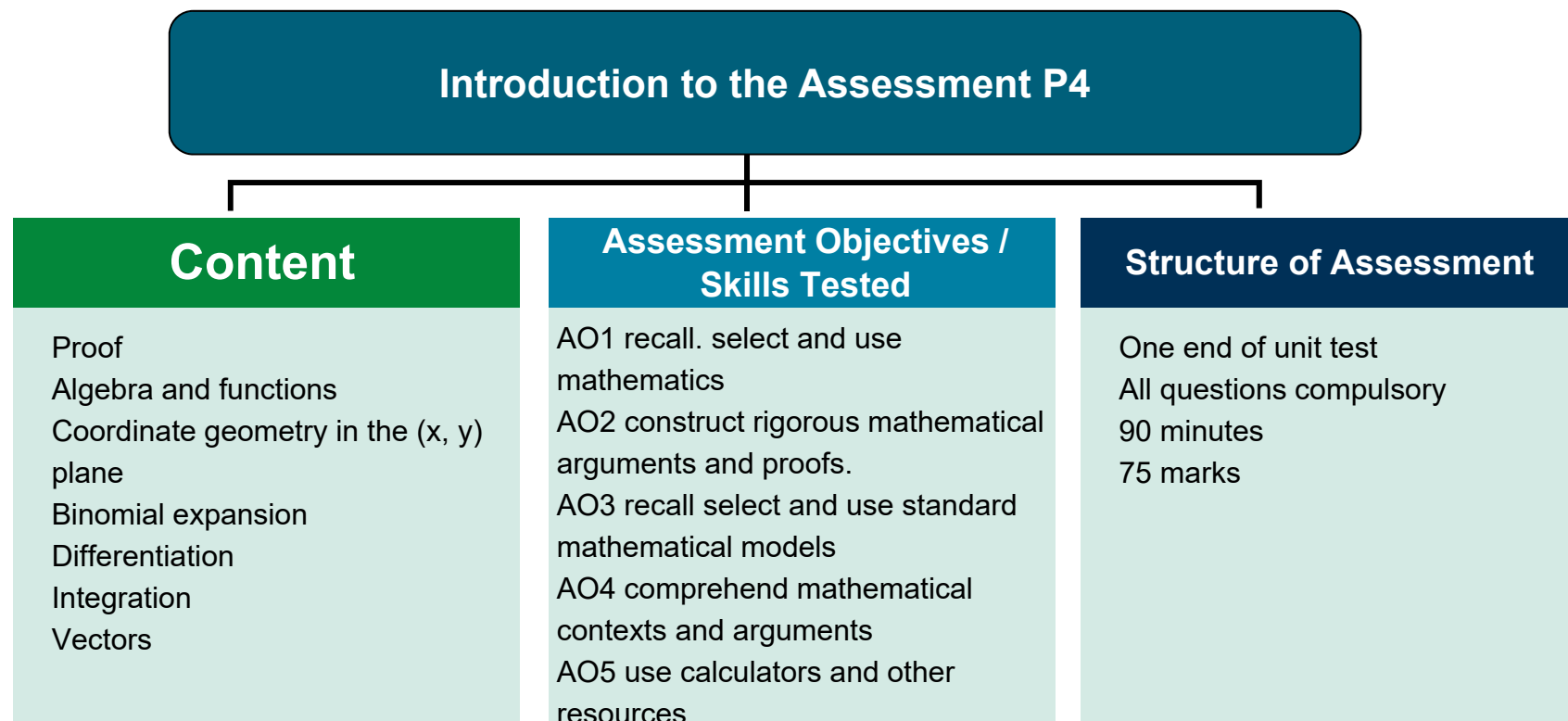
With reference to the model, interpret the value of the constant a

What is the interpretation of the gradient in this case?



P4





P4 assumes knowledge of P1, P2 and P3

P4 Content

- What's new - NOTHING!

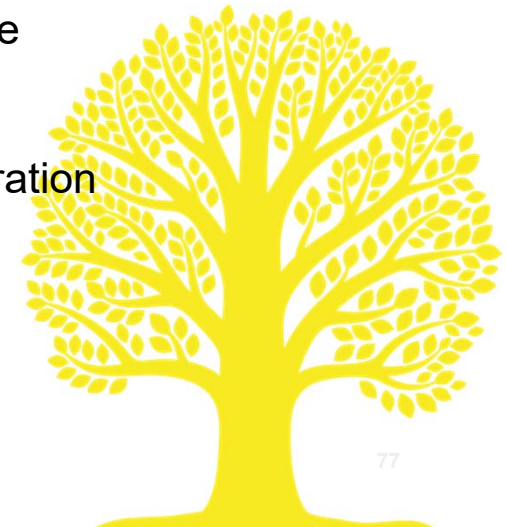
There is no new content in P4 although we have made some more explicit statements than in C34 :

1.1 Proof by contradiction – including the irrationality of $\sqrt{2}$ and the infinity of primes

6.2 Simple cases of integration by substitution where the student has to decide on and use a suitable substitution

6.2 Simple cases of integration by parts where more than one application may be required,

6.3 Simple cases of integration using partial fractions – integration of other rational expressions is also required (not just in P4)



P4 Content

<http://www.numberempire.com/numberfactorizer.php>

- Exploring the proof section
- 1.1 Proof by contradiction –the infinity of primes.

Euclid's proof of the infinity of primes.

The idea is to consider the sequence which starts

$$2 \quad 2 + 1 \quad 2 \times 3 + 1 \quad 2 \times 3 \times 5 + 1 \quad 2 \times 3 \times 5 \times 7 + 1$$

with values

$$2 \quad 3 \quad 7 \quad 31 \quad 211 \quad \text{which are all prime}$$

so is

$$2 \times 3 \times 5 \times 7 \times 11 + 1 = 2311$$

But $2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031$ is not prime ($= 59 \times 509$)



P4 Content

- Exploring the proof section

1.1 Proof by contradiction – the infinity of primes.

We need an economical way of representing primes.

The sensible way is to let p_n be the n th and final prime.

Then, as in the previous slide let $N = 2 \times 3 \times 5 \times \dots \times p_n + 1$

None of the primes 2, 3, 5, ..., p_n will divide into N exactly because N will give a remainder of 1 when divided by any of them.

But thinking of the case $2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031 (= 59 \times 509)$

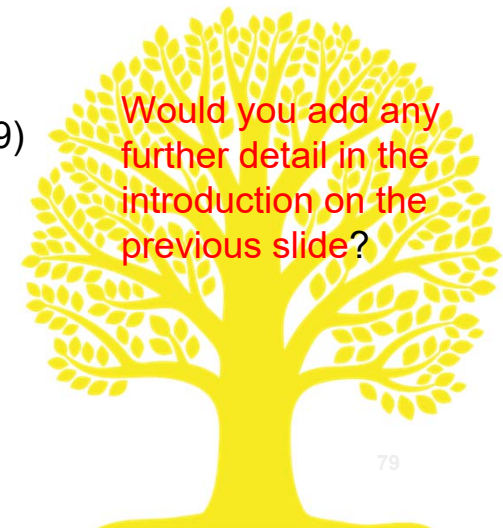
So we cannot claim N is prime.

However,.....

So p_n is the largest prime number.

The product consists of all the primes.

Would you add any further detail in the introduction on the previous slide?



P4 Content

- Exploring the proof section
From the specimen paper

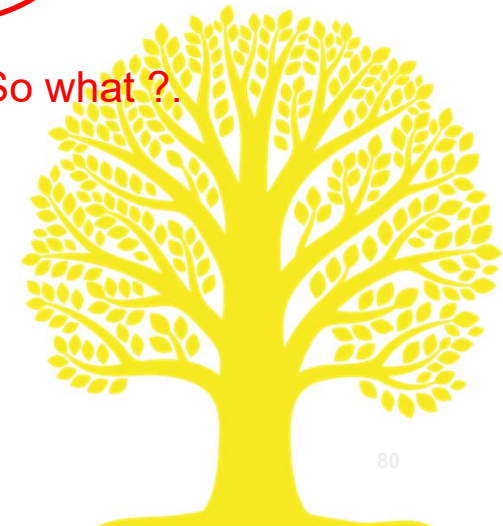
Although students will have met rationalising denominators they may need to be taught about what an irrational number is.

6. Prove by contradiction that, if a, b are positive real numbers, then $a + b \geq 2\sqrt{ab}$

Suppose to the contrary $a + b < 2\sqrt{ab}$

Squaring both sides and collecting terms gives $(a - b)^2 < 0$

So what ?



P4 Content

- Exploring integration

6.2 **Simple** cases of integration by substitution where the student has to decide on and use a suitable substitution

Find $I = \int x^2 \sqrt{2x+1} \, dx$

$$u = 2x + 1 \quad du = 2dx$$

$$I = \int x^2 \sqrt{u} \frac{1}{2} du$$

$$x = \frac{(u-1)}{2}$$

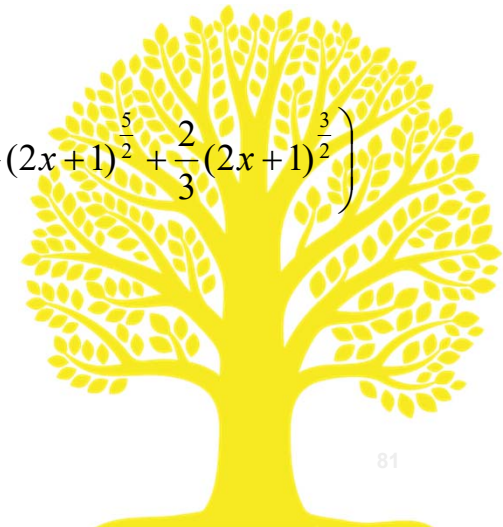
When in the evaluation should we do this?

$$I = \int \frac{(u-1)^2}{8} u^{\frac{1}{2}} du = \frac{1}{8} \int u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}} du = \frac{1}{8} \left(\frac{2}{7} (2x+1)^{\frac{7}{2}} - \frac{4}{5} (2x+1)^{\frac{5}{2}} + \frac{2}{3} (2x+1)^{\frac{3}{2}} \right)$$

Heuristic: when faced with a product substitute for the more complicated one



Do we have any comments on this?



P4 Content

- Exploring integration

6.2 **Simple** cases of integration by substitution where the student has to decide on and use a suitable substitution

Find the exact value of $\int_{-1}^2 \frac{4x}{\sqrt{2x+1}} dx$

Originally I had $x + 1$ under the square root.

Suggest a reason why I changed it.



P4 Content

- Exploring integration

6.2 **Simple** cases of integration by substitution where the student has to decide on and use a suitable substitution

Find the exact value of $I = \int_0^4 \frac{4x}{\sqrt{2x+1}} dx$

Put $u = 2x + 1$

$$du = 2dx$$

New limits are 1 and 9

$$I = \int_1^9 \frac{4x}{\sqrt{u}} \frac{1}{2} du = \int_1^9 \frac{u-1}{\sqrt{u}} du$$

$$I = \int_1^9 u^{\frac{1}{2}} - u^{-\frac{1}{2}} du$$

$$I = \left[\frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right]_1^9 = \frac{40}{3}$$



P4 Content

- Exploring integration

6.2 **Simple** cases of integration by substitution where the student has to decide on and use a suitable substitution

Find the exact value of $I = \int_0^4 \frac{4x}{\sqrt{2x+1}} dx$

Put $u = 2x + 1$ **B1 for a suitable substitution**
 $du = 2dx$

New limits are 1 and 9

$$I = \int_1^9 \frac{4x}{\sqrt{u}} \frac{1}{2} du = \int_1^9 \frac{u-1}{\sqrt{u}} du$$

M1 for using $du/dx = 2$ and an integrand a function of u only

$$I = \int_1^9 u^{\frac{1}{2}} - u^{-\frac{1}{2}} du$$

M1 for a form which can readily be integrated

$$I = \left[\frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right]_1^9 = \frac{40}{3}$$

M1 (dep) for an integrated form and new limits (or equivalent)

A1



P4 Content

Integrals work sheet

Find some time after this session to spend a few minutes working through the integrals sheet

Make any notes about what issues arose.



P4 Content

Activity 4

There several questions based on P4 (some are proofs!).
You will need your copy of the specification.

Please think about the following:

Which sections of the specification do students need to know to do the question?

Is the question suitable (with rigorous wording etc) for a written examination?

Is the question better suited to classroom use?

Avoid this question!

I'm going to talk about Q5 shortly so
you might want to look at it first



P4 Content

Activity 4

Q5 Prove, by contradiction, that no power of 2 can be written as the sum of consecutive whole numbers.

5 Suppose it can.

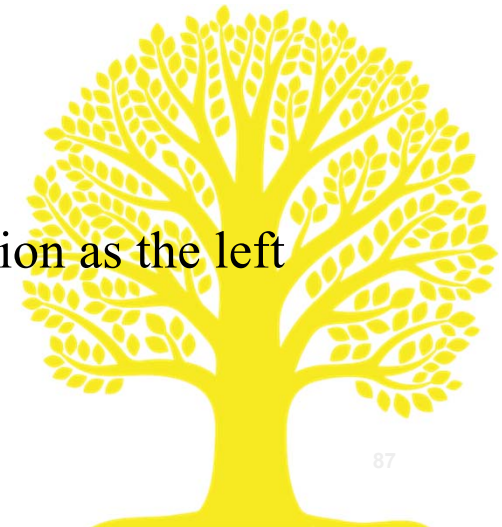
$$2^k = n + n + 1 + \dots + n + r - 1 \text{ where } r > 1$$

Using sum of an arithmetic series $2^k = n + n + 1 + \dots + n + r - 1 = \frac{r}{2}(2n + r - 1)$

$$2^{k+1} = r(2n + r - 1) \quad (*)$$

So r must be a power of 2, say 2^s ($k + 1 > s > 0$)

(*) then becomes $2^{k+1-s} = 2n + 2^s - 1$ which is a contradiction as the left hand side is even but the right hand side is odd



D₁



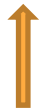
D1 Content

What's new?

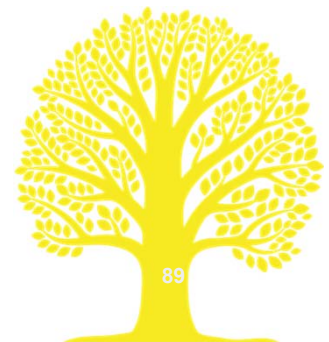
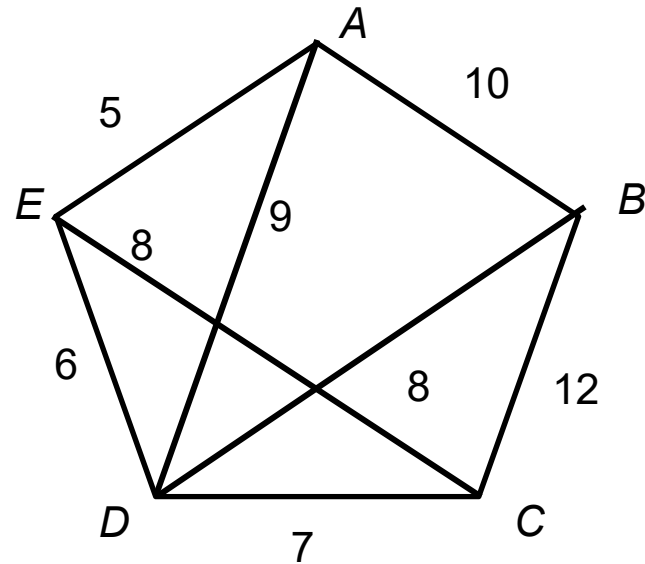
The Travelling Salesman Problem (TSP)

Given a connected graph with distances on all arcs, find the tour of the graph which has minimum length

What is the length of the shortest tour that starts and ends at A?



Each vertex must be visited at least once



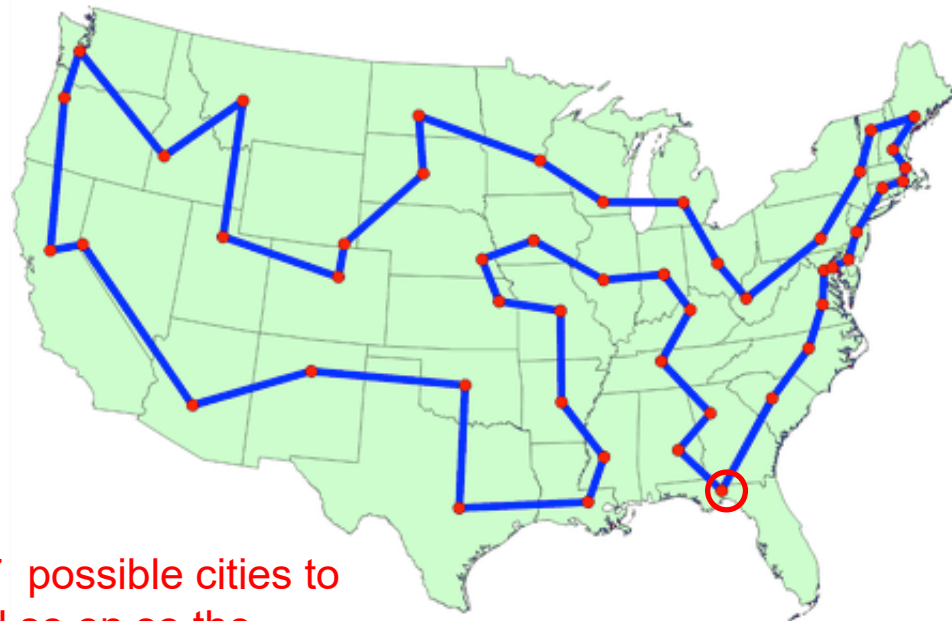
D1 Content

What's new?

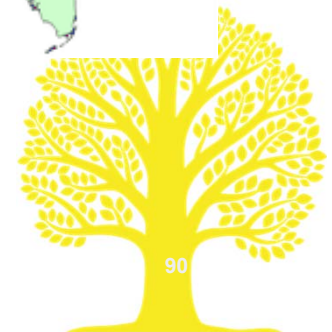
The Travelling Salesman Problem (TSP)

This is a tour of 48 state capitals, one in each state, in continental USA

TSP is a good example of showing why 'brute force' (enumeration) cannot always work.



If I start from Tallahassee, there are 47 possible cities to visit next. From the second city 46 and so on so the total number of tours is $(2 \times 47!)$ or about 5×10^{59}



D1 Content

What's new?

Is 5×10^{59} a 'big' number?

A state of the art processor can perform about 3×10^{11} instructions per second

$(48 \times 5 \times 10^{59}) \div (3 \times 10^{11})$ is about 8×10^{49} seconds



D1 Content

To keep the volume of content equivalent in the new course the section on Matching in the current course has been deleted.



Considering Delivery Strategies and sharing best practice

- 
1. Teaching Strategies.
 2. Resources.
 3. Technology.

New IAL Content

We hope you agree with us in feeling that the changes made to the content are manageable

That the scheme of assessment has the flexibility you like.

That you have sufficient resources to deliver the course to a high standard.

It is sufficiently challenging at the top end but accessible at the bottom end



Support Overview

Free Support

Getting Started
Guide & Scheme of
Work

Getting Ready to
Teach Events

Subject
interpretation of
transferable skills

Subject Advisor

Results Plus

Regional Support
Manager

Additional support for selected subjects

**Curriculum
Matched
Publishing**

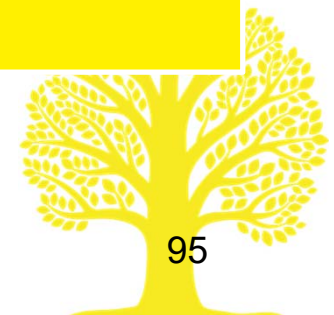
Lesson plans

Exemplar Marked
Responses

Topic booklets &
Subject guides

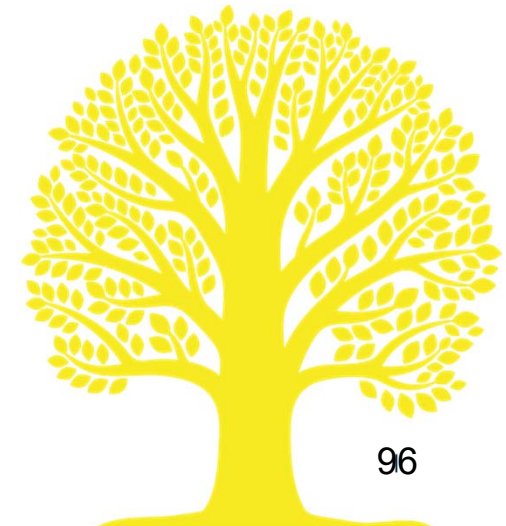
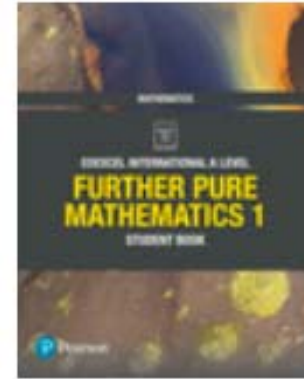
Additional SAMs

Exam Wizard



Current Published Materials

- Please see the Published Resources web-page for available resources, including:
 - Student books
 - Teacher Resource books



Other useful links

[1. Grade Boundaries](#)

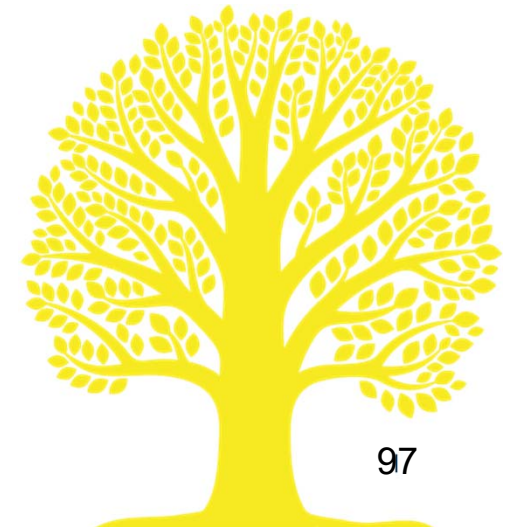
This page shows the minimum marks needed to achieve a certain grade for all UK and international examinations. Also refer to the examiners report which is available for download with other documents.

[2. Examination Results Statistics](#)

Results statistics summarise the overall grade outcomes of candidates sitting Pearson Edexcel examinations.

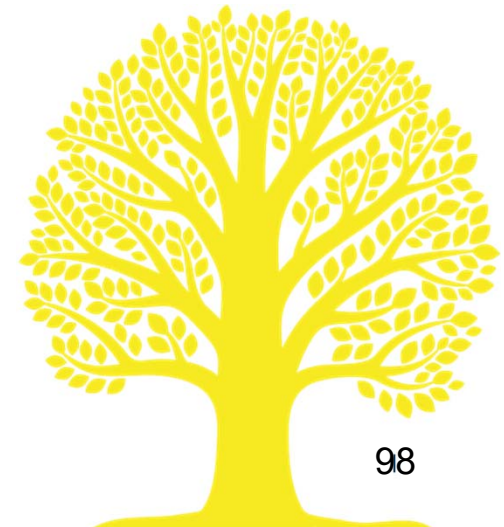
[3. Progress to University](#)

Here you can find information and guidance about how to progress to universities worldwide with Pearson Edexcel qualifications.

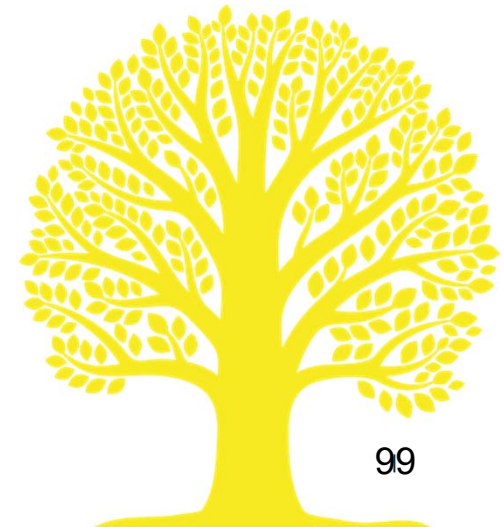


- Free online results analysis tool for teachers
- Provides a detailed breakdown of student performance in Edexcel exams.
- Identify topics and questions where the student could benefit from further learning
- Use this knowledge to inform teaching strategies and approaches
- Provides a comparison of student performance at regional level.
- Allows centres to view their country's results compared to the total Edexcel cohort.
- Mock exams results can also be fed into the system to produce an analysis
- Schools can sign up for free ResultsPlus account in just a few quick and easy steps:

<https://qualifications.pearson.com/en/support/Services/ResultsPlus.html>



- Free tool for teachers containing a bank of past paper questions to help create their own bespoke mock exams and tests to focus on particular topic areas as needed
- Use existing mark schemes for accurate marking
- Use existing examiner report for insight
- Use the results to understand where students need more support, informing teaching strategies.



Contact your dedicated Subject Advisor

Subject Advisor details

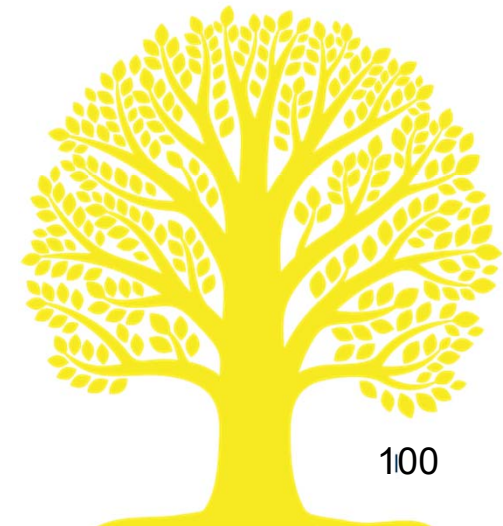
Your subject advisor is **Graham Cumming**

Phone: **+44 (0)20 7010 2174**

Twitter: **@EmporiumMaths**

Email: TeachingMaths@pearson.com

Sign up for monthly newsletters from Graham to stay on top of qualification updates, training, course materials and industry news.



Any questions?

**Thank you for
attending this event.**

How did we do?

*Please fill in the evaluation form that you'll
receive via e-mail in a few minutes.*

There's so much more to learn

Find out more about our range of events at
<http://qualifications.pearson.com/training>

ALWAYS LEARNING